NASA TECHNICAL TRANSLATION

NASA TT F-521

NASA TT F-521

C. 1

TECH LIBRARY KAFB, NM
0069026

LOAN COPY: RETURN TO
AFWL (WLIL-2)
KIRTLAND AFB, N MEX

# MAGNETOBREMS (SYNCHROTRON) EMISSION AND REABSORPTION

*by V. L. Ginzburg, V. N. Sazonov, and S. I. Syrovatskiy*

*Preprint* No. 83 *of the Lebedev Physical Institute*
*Moscow,* 1967

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1968

TECH LIBRARY KAFB, NM
0069026
NASA TT F-521

# MAGNETOBREMS (SYNCHROTRON) EMISSION AND REABSORPTION

By V. L. Ginzburg, V. N. Sazonov, and S. I. Syrovatskiy

Translation of "O magnitotormoznom (Sinkhrotronnom) izluchenii i yego reabsorbtsii."
Preprint No. 83 of the Lebedev Physical Institute, Moscow, 1967.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – CFSTI price $3.00

CUT ALONG THIS LINE

FOLD LINE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546

OFFICIAL BUSINESS

POSTAGE AND FEES PAID
NATIONAL AERONAUTICS & SPACE ADMINISTRATION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
CODE USS-T
WASHINGTON, D.C. 20546

NASA TTF No.
521

FOLD LINE

CUT ALONG THIS LINE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

# TECHNICAL TRANSLATION EVALUATION

*Budget Bureau No. 104-R037*
*Approval Expires: Sept. 30, 1969*

TO: THE USERS OF THIS TRANSLATION

NASA TTF NO. 521

MAINTAINING THE QUALITY OF NASA TRANSLATIONS REQUIRES A CONTINUOUS EVALUATION PROGRAM. PLEASE COMPLETE AND MAIL THIS FORM TO AID IN THE EVALUATION OF THE USEFULNESS AND QUALITY OF THE TRANSLATING SERVICE.

**THIS PUBLICATION** *(Check one or more)*

- ☐ FURNISHED VALUABLE NEW DATA OR A NEW APPROACH TO RESEARCH.
- ☐ VERIFIED INFORMATION AVAILABLE FROM OTHER SOURCES.
- ☐ FURNISHED INTERESTING BACKGROUND INFORMATION.
- ☐ OTHER *(Explain)*: _____

FOLD LINE FOLD LINE

**TRANSLATION TEXT** *(Check one)*

- ☐ IS TECHNICALLY ACCURATE.
- ☐ IS SUFFICIENTLY ACCURATE FOR OUR PURPOSE.
- ☐ IS SATISFACTORY, BUT CONTAINS MINOR ERRORS.
- ☐ IS UNSATISFACTORY BECAUSE OF *(Check one or more)*:
  - ☐ POOR TERMINOLOGY.
  - ☐ INCOMPLETE TRANSLATION.
  - ☐ NUMERICAL INACCURACIES.
  - ☐ ILLEGIBLE SYMBOLS, TABULATIONS, OR CURVES.
  - ☐ OTHER *(Explain)*:

_____

FOLD LINE FOLD LINE

REMARKS

FROM

DATE

**NOTE:** REMOVE THIS SHEET FROM THE PUBLICATION, FOLD AS INDICATED, STAPLE OR TAPE, AND MAIL. NO POSTAGE NECESSARY.

**NASA Form 1332** AUG 66

GPO 915-777

CUT ALONG THIS LINE

CUT ALONG THIS LINE

# MAGNETOBREMS (SYNCHROTRON) EMISSION AND REABSORPTION /1*

V.L.Ginzburg, V.N.Sazonov, and S.I.Syrovatskiy[1)]

ABSTRACT. The problem of magnetic-acceleration (synchrotron) emission at helical motion of relativistic electrons emitting from a source with relativistic velocity is discussed and mathematically formulated. Problems of reabsorption of magnetobrems emission, with respect to radio emission by quasars, are reviewed, followed by a study of the motion of particles in a magnetic field in the presence of losses. Special emphasis is placed on this motion for particle aggregates of high energy density and the resultant changes in the magnetic field, with formulas for the change in energy of the particle group. Errors in formulas and conclusions by other authors are analyzed.

CONTENTS

1. Magnetobrems Emission for the Helical Motion of a Particle (Elementary Treatment)
2. Čerenkov Radiation for a Particle Crossing a Sheet of Matter (Plate)
3. Magnetobrems Emission for a Discrete Particle Moving along a Helical Trajectory
4. Emission for a System of Particles
5. Reabsorption in the Ultrarelativistic Case
6. Magnetic Field Change due to Magnetobrems Emission (Losses)
References

## Introduction

Magnetobrems (synchrotron) emission has been studied for many years in the literature and its corresponding formulas are widely used, especially in astrophysics (Ref.1, 2). Thus, it is surprising that in recent years new ideas in this area have been presented. Moreover, it seems that the authors of many articles have cited, and occasionally used, erroneous or inaccurate formulas. These refer to: 1) The case of emission of particles that move in a magnetic /2 field along a helical, not circular, trajectory (in other words, we are talking here about the motion of particles with velocities $\vec{v}$ that make an angle $\theta \neq \neq \frac{\pi}{2}$ with the magnetic field $\vec{H}$). In practice, formulas for the intensity of

[1)] P.N.Lebedev Physical Institute, Academy of Sciences USSR, Moscow.

* Numbers in the margin indicate pagination in the foreign text.

the synchrotron emission of one electron moving at an angle of $\theta \neq \frac{\pi}{2}$ have not been interpreted correctly, and the statements made by various authors (Ref.3-5) are erroneous. Moreover, the expressions for the intensity of the magnetic-acceleration emission of an aggregate of particles, which are used in radio-astronomy (Ref.1, 2), are based on formulas for the intensity of emission of a discrete electron. Fortunately, the inaccuracy* of these formulas (Ref.3 - 5) is not expressed in the transition to the intensity of emission of an aggregate or group of electrons located in a fixed region (e.g., in the envelope of a supernova star). Therefore, all applications of the magnetobrems theory formulas by Ginzburg (Ref.1, 2) and other authors are correct. However, if we were to /3 consider the magnetobrems emission of a cloud of relativistic electrons emitting from a certain source and moving with relativistic velocity, then the application of some of the formulas by Ginzburg (Ref.1, 2) would lead to significant errors.

The goal of our work is to study this problem of magnetobrems emission in the case of helical motion, a problem which has both practical and methodological significance. In addition, we will consider the problem of the reabsorption of magnetobrems emission, which has recently attracted so much attention (Ref.8-10), especially in connection with the study of the radio emission of quasars. In addition, the problem was not studied in sufficient detail by Ginzburg (Ref.1, 2). In our work, we thought it expedient to include another chapter (Chapt.6) devoted to an examination of the motion of particles in a magnetic field in the presence of losses and under consideration of their interaction with the source creating the field (e.g., "solenoid"). No unexpected results were obtained, but from a methodological point of view, the problem is very interesting. We believe that this problem will be of practical importance when dealing with an aggregate of particles of rather high energy density.

## 1. Elementary Study of Emission at a Charge Moving along a Helical Trajectory /4

In practice, an ultrarelativistic electron**(we will be concerned only with such a case) emits only in the direction of its instantaneous velocity or, more exactly, in a cone of angle

$$\psi \sim \frac{mc^2}{E}, \quad E \gg mc^2. \tag{1.1}$$

---

* According to our information, this circumstance (inaccuracy of some of the expressions used when $\theta \neq \frac{\pi}{2}$) has been noticed, independently of us and partially even before us, by others (V.A.Razin, E.G.Mychelkin, G.B.Field, and J.Skargle). Moreover, correct general formulas for the emission of a discrete particle (Ref.6, 7) have been cited and interpreted for the ultrarelativistic case.

** We will talk here specifically about electrons. Of course, all designations refer to particles with a charge e and a rest mass m.

In a subsequent qualitative analysis, we will assume (whenever possible) the radiation to be needle-shaped, i.e., the angle $\psi$ is taken as small as desired. When traveling in a constant and homogeneous magnetic field with an intensity $\vec{H}$, the electron, generally speaking, moves along a helical trajectory with a velocity $v_{\parallel} = v \cos\theta$ along the field and with a velocity of $v_{\perp} = v \sin\theta$ across the field (of course, the total velocity is $v = \sqrt{v_{\parallel}^2 + v_{\perp}^2}$). The cyclotron frequency $\omega_H$ depends only on v and is equal to

$$\omega_H = \frac{eH}{mc}\sqrt{1-\frac{v^2}{c^2}} = \frac{eH}{mc}\,\frac{mc^2}{E}. \qquad (1.2)$$

If the motion is circular (i.e., $v_{\parallel} = 0$, $v_{\perp} = v$), then, in view of what has been said, an ultrarelativistic electron emits only in the orbital plane. An observer (recording instrument) located in this plane "will see" emission bursts when the electron travels exactly in his direction (in the case of a vacuum, it is also necessary to consider the delay time of the emission propagation as /5 equal to $\frac{\tau}{c}$ where $\tau$ is the distance from the electron to the observer). Obviously, the bursts will be repeated each period or, in other words, will follow each other in sequence within a time $\tau = \frac{2\pi}{\omega_H} = \frac{2\pi mc}{eH} \cdot \frac{E}{mc^2}$. As distinctly shown by Ginzburg (Ref.1, 2), the characteristic duration of each burst under the condition (1.2) is of the order of $\Delta t \sim \frac{mc}{eH}\left(\frac{mc^2}{E}\right)^2$ and the observer registers the field outlined in Fig.1. It is obvious that the expansion of this field in a Fourier series leads to a spectrum consisting of frequency harmonics of $\omega_H$. All corresponding expressions for the field strength, intensity, and other quantities given by other authors (Ref.1 - 5) are correct, and we need not dwell on them. As stated before, the formulas for noncircular motion, at a longitudinal velocity of $v_{\parallel} = v\cos\theta \neq 0$ (i.e., at an angle $\theta \neq \frac{\pi}{2}$) are incorrect. The error source is obvious from the original expression by Westfold (Ref.5) for the emission field strength, which is written in the form

$$\vec{\mathcal{E}} = \mathcal{R}e\left(\sum_{n=1}^{\infty} \vec{\mathcal{E}}_n e^{-in\omega_H t}\right). \qquad (1.3)$$

In fact, at $v_{\parallel} \neq 0$ the emission pulses do not follow one another within a time $\tau = \frac{2\pi}{\omega_H}$ but within a time $\tau^*$ which differs from $\tau$ due to the Doppler effect.

Using Fig.2, it is easy to find the time $\tau^*$ in an elementary manner. For the observer selected, the emission bursts occur when an electron is located at the points A, B, C ... (for simplicity, we will assume now and later that the emission is strictly needle-shaped). In other words, the electron "looks" /6

toward the observer at these points. The time interval between the instants at which the electron passes through the points A and B is equal to the period $\tau = \frac{2\pi}{\omega_H}$. The distance between the points A and B is $v_{\parallel}\tau = v\tau \cos\theta$ (where $\theta$ is the angle between $\vec{v}$ and $\vec{H}$), and the pulse emitted at the point A during this time will cover the path $c\tau$. It is clear from Fig.2 that the pulse, emitted at the point B, will arrive at the observer with a delay time from the first pulse of

$$\tau^* = \tau\left(1 - \frac{v_{\parallel}\cos\theta}{c}\right) = \tau\left(1 - \frac{v}{c}\cos^2\theta\right) \simeq \tau\sin^2\theta = \frac{2\pi}{\omega_H}\sin^2\theta, \qquad (1.4)$$

where we assume, in passing to the expression before the last, that the complete calculation is carried out for the limiting case $v \to c$. It should be noted that the pattern in which the emission reaches the observer in the form of discrete pulses holds only when $\theta \gg \psi \sim \frac{mc^2}{E}$. The fact that the expression $\tau^* = \tau\left(1 - \frac{v_{\parallel}\cos\theta}{c}\right)$ is general in character is connected neither with the assumption of the "needle-like shape" of the emission nor with the possibility of separating it into discrete pulses (moreover, $v_{\parallel}\cos\theta$ is replaced by $v_{\parallel}\cos\theta'$; see Section 3).

Thus, the nonrelativistic electron field in the wave zone consists of the frequency harmonics

$$\omega_H^* = \frac{q\pi}{\tau^*} = \frac{\omega_H}{\sin^2\theta}. \qquad (1.5)$$

If we assume that, in the cases of interest here, the harmonics are not resolved, then the above fact is not too significant in itself. Ginzburg's (Ref.1, 2) determination of the pulse width $\Delta t \sim \frac{mc}{eH_{\perp}}\left(\frac{mc^2}{E}\right)^2$ and of the characteristic frequency is completely correct (here and later, $H_{\perp} = H\sin\theta$). /7 However, the change in pulse interval manifests itself not only in the spectrum but also in all characteristics of the emission field, especially in its intensity which is fixed at the point of the observer. Actually, if an electron for each revolution $\left(\text{for the time } \tau = \frac{2\pi}{\omega_H}\right)$ were to lose an energy $\Delta E = P\tau$ because of emission, then - in view of the above statements - it would be obvious that this energy proceeds for a time $\tau^*$ toward the "observers" stationed on a given fixed sphere at a distance $r$ from the electron; therefore, the average observable emissive power (total energy flux) will be equal to

$$P^* = \frac{\Delta E}{\tau^*} = \frac{P\tau}{\tau^*} = \frac{P}{\sin^2\theta}. \qquad (1.6)$$

At first glance, it would seem that the law of conservation of energy is violated. In unit time, the electron loses an energy P [the value of P is determined from a well-known formula; see Sect.4.1 and (Ref.2)]. All of this energy is converted into emission and thus should be equal to the total radiant flux striking the sphere under study. Often, the radiative losses of a particle are calculated and then equated to the total radiant flux. In addition, in the stationary case and for an emitter, radiative losses can be treated in this manner. In general, the work done by the emitter in unit time (lost power P) is equal to the total flux through a given surface plus the change in the field energy $\frac{\partial}{\partial t}\int\left(\frac{E^2+H^2}{8\pi}\right)dv$ in the volume enclosed by this surface. In the /8 case of interest here, the space occupied by the radiation and located between the moving electron and the surface fixed in the plane in which the observations take place is decreasing steadily. The energy contained in this area also decreases so that the power of the assumed emission $P^*$ is greater than the power loss P. At the same time (Ref.4), the power loss P was used in converting to spectral magnitudes. In calculating the displacement of the emitter, this approach cannot lead to correct expressions for the radiation intensity which is fixed to the given stationary surface. Moreover, if the emitting particles are in a fixed volume (for example, the envelope of a supernova star) or, more accurately, if the distribution function of the emitting particles does not change in time, then the emission intensity of the particle aggregate coincides with the spectral power loss. This conclusion is obvious from the law of conservation of energy and can be confirmed by direct calculations (Sect.4).

Since this elementary problem had not been studied for such a long time and since it often led scientists (us included) to use formulas which were not always and not completely correct, our attempt to present the material contained in this article in a somewhat similar form seems justified.

## 2. Čerenkov Radiation for a Particle Passing through a Plate (Layer of Matter) /9

The observation that the radiant flux for a moving source does not coincide with the energy losses in unit time, has a very general character. As an illustration in addition to magnetobrems emission, let us consider a Čerenkov radiant flux produced when a particle passes through a plate (Fig.3). For simplicity, let us assume the medium as being non-dispersive or, as one chooses, let us limit ourselves to a frequency range for which the dispersion is small, so that the group velocity of light is equal to the phase velocity c/n. Let us designate by $\left(\frac{dE}{dx}\right)_c$ the energy losses (referred to unit length) to the Čerenkov radiation in the frequency range under study. A particle with a velocity v will pass in the time t = d/v through a plate of thickness d, after having lost an energy $\left(\frac{dE}{dx}\right)_c \cdot d$. The losses referred to unit time are equal to $\left(\frac{dE}{dx}\right)_c \cdot v =$ = P. The particle begins to emit upon entering the plate (instant t = 0) and emits for the entire time t = d/v until it leaves the plate. From then on, the

radiation will be "located" on the surface of the cone with an angle $\pi - 2\theta$, where $\cos\theta = \frac{c}{nv}$ (dispersion is neglected). It is clear from Fig.3 that light begins to leave the plate at the instant $t = d/v$ and that the bursts end as soon as the point A (the rear portion of the cone emitted at the instant $t = 0$) reaches the point B. It is obvious that the distance $\theta A = \frac{c}{n} t = \frac{c}{n} \frac{d}{v} =$ $= d \cos\theta$ and the distance $\theta B = d/\cos\theta$. Hence $AB = d\left(\frac{1}{\cos\theta} - \cos\theta\right) =$ $= d \frac{\sin^2\theta}{\cos\theta}$ and the duration of the burst of Čerenkov luminescence* will be /10

$$t^* = \frac{d\sin^2\theta}{\cos\theta \cdot c/n} = t \cdot \tan^2\theta \qquad (2.1)$$

since $d = vt$ and $\cos\theta = \frac{c}{nv}$. Therefore, the total radiant flux in the burst is

$$P^* = \frac{(dE/dx)_c \cdot d}{t \cdot \tan^2\theta} = \frac{P}{\tan^2\theta} = P\cot^2\theta \qquad (2.2)$$

where P is the energy loss in unit time (reflection from the surface of the plate is neglected). Thus, this flux - generally speaking - is not equal to the energy losses in unit time.

## 3. Magnetobrems Emission of a Discrete Particle Moving at an Arbitrary Angle to the Field /11

Let us consider a system of coordinates corresponding to Fig.4 such that the axis $\vec{e}_3$ is directed along the external magnetic field $\vec{H} = \vec{H}e_3$. Let a particle with a charge e move in a field $\vec{H}$ along a trajectory

$$\vec{r}_0(t) = \frac{c\beta_\perp}{\omega_H}\left\{-\vec{e}_1\cos\omega_H t + \vec{e}_2\sin\omega_H t\right\} + \vec{e}_3 c\beta_\parallel t$$
$$\vec{\beta}(t) = \beta_\perp\left\{\vec{e}_1\sin\omega_H t + \vec{e}_2\cos\omega_H t\right\} + \vec{e}_3\beta_\parallel . \qquad (3.1)$$

Here, $\vec{\beta}$ is the velocity of the particle in light velocity units c, while $\beta_\parallel$ and $\beta_\perp$ are the values of its projection in the direction of the field and across it; the quantity $\omega_H$ is determined from eq.(1.2). For a negatively charged particle, $\omega_H < 0$. Figure 4 shows the trajectory of a negatively charged particle (for example, an electron).

At a great distance from the particle, in the wave zone, the Fourier components of the vector potential and the particle electric field strength are

---

* The question of the duration of a Čerenkov burst, especially with respect to its application to Čerenkov counters, is studied elsewhere (Ref.11).

correspondingly equal to [see (Ref.12, Sect.66)]

$$\vec{A}_\omega = e^{i\frac{\omega}{c}\vec{\kappa}\vec{r}}\frac{e}{2\pi r}\int_{-\infty}^{+\infty}\vec{\beta}(t)e^{i\left\{\omega t-\frac{\omega}{c}\vec{\kappa}\vec{r}_0(t)\right\}}dt,$$

$$\vec{\varepsilon}_\omega = i\frac{\omega}{c}[\vec{\kappa}[\vec{A}_\omega\vec{\kappa}]] = i\frac{\omega}{c}\left(\vec{A}_\omega - \vec{\kappa}\left(\vec{A}_\omega\vec{\kappa}\right)\right), \tag{3.2}$$

where $\vec{K}$ is a single vector in the direction of emission (from the particle to the observer) and $\vec{r} = r\vec{K}$ where r is the distance between the observer and the position of the electron at a given fixed instant of time; moreover, we assume that the vector $\vec{K}$ is located in the plane $(\vec{e}_2, \vec{e}_3)$ and makes an angle $\theta'$ with the magnetic field, i.e., $\vec{K} = \{0, K_2, K_3\} = \{0, \sin\theta', \cos\theta'\}$. The angle between $\vec{v}$ and $\vec{H}$ is designated by $\theta$ (Fig.4). /12

As indicated in eq.(3.2), only the velocity component crossing the path of emission enters the expression for $\vec{\varepsilon}_\omega$

$$\vec{\beta}_t = \vec{\beta} - \vec{\kappa}\left(\vec{\beta}\vec{\kappa}\right) = \vec{e}_1\beta_\perp\sin\omega_H t + \left(\vec{e}_2K_3 - \vec{e}_3K_2\right)\left(\beta_\perp K_3\cos\omega_H t - \beta_\parallel K_2\right). \tag{3.3}$$

It is convenient to introduce the triplet of unitary vectors $\vec{l}_1$, $\vec{l}_2$, $\vec{K}$ such that

$$\vec{\ell}_2 = \vec{e}_3K_2 - \vec{e}_2K_3, \quad \vec{\ell}_1 = [\ell_2\cdot\vec{\kappa}] = -\vec{e}_1. \tag{3.4}$$

The vector $\vec{l}_2$ is directed along the projection $\vec{H}$ onto the plane perpendicular to the direction of observation (plane of the paper), i.e., along the vector $\vec{e}_3 - \vec{K}(\vec{e}_3 \cdot K)$.

From eqs.(3.2), (3.3), and (3.4), we obtain

$$\vec{\varepsilon}_\omega = e^{i\frac{\omega}{c}r}\frac{ie\omega}{2\pi cr}\int_{-\infty}^{+\infty}\vec{\beta}_t(t)e^{i\left(\omega t-\frac{\omega}{c}\vec{\kappa}\vec{r}_0(t)\right)}dt \tag{3.5}$$

where /13

$$\vec{\beta}_t(t) = -\vec{\ell}_1\beta_\perp\sin\omega_H t - \vec{\ell}_2\left(\beta_\perp K_3\cos\omega_H t - \beta_\parallel K_2\right). \tag{3.6}$$

To calculate the integral (3.5), we should note that the index of the integrand exponent in eq.(3.5) is equal to [see eq.(3.1)]

$$\omega t - \frac{\omega}{c}\vec{\kappa}\vec{r}_0(t) = (1-\beta_{\parallel}\kappa_3)\omega t - z\sin\omega_H t, \tag{3.7}$$

where

$$z = \frac{\omega}{\omega_H}\beta_{\perp}\kappa_{\perp}. \tag{3.8}$$

Furthermore, let us use the expression

$$e^{-iz\sin\omega_H t} = \sum_{n=-\infty}^{\infty} J_n(z)\, e^{-in\omega_H t}, \tag{3.9}$$

where $J_n(z)$ is a Bessel function of the first kind.

Integrating with respect to t leads to the appearance of δ-functions:

$$\vec{\mathcal{E}}_{\omega} = e^{i\frac{\omega}{c}} \frac{e\omega}{cr} \sum_{n=-\infty}^{\infty} \left\{ \vec{e}_1 \beta_{\perp} J_n'(z) - \right.$$

$$\left. - i\vec{e}_2 \left(\beta_{\perp}\kappa_3 \frac{n}{z} - \beta_{\parallel}\kappa_{\perp}\right) J_n(z) \right\} \delta\left\{(1-\beta_{\parallel}\kappa_3)\omega - n\omega_H\right\}. \tag{3.10}$$

Thus, the emission has a discrete spectrum with the frequencies:

$$\omega = \omega_n = n\frac{\omega_H}{1-\beta_{\parallel}\kappa_3} = \frac{\omega_H}{1-\beta\cos\theta\cos\theta'}. \tag{3.11}$$

In the ultrarelativistic case, $\beta \to 1$ and the emission, in practice, is directed /14 along the instantaneous velocity of the particle, i.e., the angle $\theta \simeq \theta'$ and $\omega_n = \frac{n\omega_H}{1 - \frac{V}{c}\cos^2\theta} = \frac{n\omega_H^*}{\sin^2\theta} + n\omega_H$, which agrees with eq.(1.5). As a result, the Fourier integral of the electric field of emission of the particle is reduced to the form

$$\vec{\mathcal{E}} = \int_{-\infty}^{+\infty} \vec{\mathcal{E}}_{\omega}\, e^{-i\omega t}\, d\omega = \mathcal{R}e \sum_{n=1}^{\infty} \vec{\mathcal{E}}_n\, e^{i\omega_n\left(\frac{z}{c} - t\right)} \tag{3.12}$$

where the amplitude of the n-th harmonic of the field is

$$\vec{\mathcal{E}}_n = \frac{2e}{cr} \frac{n\omega_H \beta\sin\theta}{(1-\beta\cos\theta\cos\theta')^2} \left\{ \vec{e}_1 J_n'(z) - i\vec{e}_2 \frac{\cos\theta' - \beta\cos\theta}{\beta\sin\theta\sin\theta'} J_n(z_n) \right\} \tag{3.13}$$

and

$$z_n = n \frac{\beta \sin\theta \sin\theta'}{1 - \beta \cos\theta \cos\theta'} . \qquad (3.14)$$

Equations (3.12) - (3.14) completely determine the emission field produced at a given rather distant point in space by the particle traveling at an arbitrary angle to the magnetic field. In what follows, it will be convenient to use the emission "polarization tensor", which is equal to

$$\tilde{\rho}_{\alpha\beta}(n) = \frac{c}{8\pi} \mathcal{E}_{n,\alpha} \mathcal{E}^{*}_{n,\beta} \qquad (3.15)$$

where $\alpha$, $\beta$ = 1, 2 and $\mathcal{E}_{n\alpha}$ is the components of the electric vector [eq.(3.13)]. /15 Moreover, the average density of the energy flux for the period (Poynting vector) in the n-th harmonics is equal to

$$\tilde{\rho}_n = Sp\, \tilde{\rho}_{\alpha\beta}(n) = \frac{c}{8\pi} \left| \vec{\mathcal{E}}_n \right|^2 . \qquad (3.16)$$

For ultrarelativistic particles for which

$$\xi = mc^2/E \ll 1 \quad , \qquad (3.17)$$

the main role is played by emission in high harmonics n $\gg$ 1, which is concentrated within the limits of a small angle.

$$\psi = \theta - \theta' \lesssim \xi = \frac{mc^2}{E} . \qquad (3.18)$$

The characteristic emission frequencies [see eqs.(1.5) or (3.11)] at $\beta \simeq 1$, $\theta \simeq \theta'$, are equal to

$$\nu = \frac{\omega}{2\pi} = \frac{n\,\omega_H}{2\pi \sin^2\theta} . \qquad (3.19)$$

For the transformation in eq.(3.13) to ultrarelativistic limits, we can use an approximate expression of Bessel functions with large values of index and argument (Ref.13, p.979). This leads to the following formula for the amplitude of the n-th harmonics of the electric emission field of an ultrarelativistic electron*:

/16

$$\vec{\mathcal{E}}_n = \frac{2e\omega_H}{\sqrt{3}\pi c r} \frac{n}{\sin^3\theta} \left\{ \vec{e}_1 (\xi^2 + \psi^2) K_{2/3}(g_n) + i \vec{e}_2 \psi (\xi^2 + \psi^2)^{1/2} K_{1/3}(g_n) \right\} \qquad (3.20)$$

* For a positively charged particle (e.g., a positron), the amplitude is complex-conjugate with respect to eq.(3.20) which refers to a direction (cont'd)

where

$$g_n = n\frac{(\xi^2+\psi^2)^{3/2}}{3\sin^3\theta} = \frac{\nu}{\nu_c}\left(1+\frac{\psi^2}{\xi^2}\right)^{3/2} \tag{3.21}$$

In the second equation of the system (3.21), we converted from the harmonics number n to the frequency $\nu$ [see eq.(3.19)] and used the notation

$$\nu_c = \frac{3\omega_H \sin\theta}{4\pi\xi^3} = \frac{3eH_\perp}{4\pi mc}\left(\frac{E}{mc^2}\right)^2. \tag{3.22}$$

In the region of high harmonics, the emission spectrum is practically continuous so that, instead of the emission polarization tensor in the n-th harmonics [eq.(3.15)], we can introduce the "spectral density polarization tensor"

$$\tilde{P}_{\alpha\beta}(\nu) = \tilde{P}_{\alpha\beta}(n)\frac{dn}{d\nu} = \frac{2\pi\sin^2\theta}{\omega_H}\tilde{P}_{\alpha\beta}(n). \tag{3.23}$$

From this and from eqs.(3.15) and (3.20), we find the spectral density of the radiant fluxes with two main directions of polarization

$$P_\nu^{(1)} = P_{11}(\nu) = \frac{3e^2\omega_H}{4\pi^2 c^2 \xi^2 \sin^2\theta}\left(\frac{\nu}{\nu_c}\right)^2\left(1+\frac{\psi^2}{\xi^2}\right)^2 K_{1/3}^2(g_\nu) \tag{3.24}$$

/17

$$P_\nu^{(2)} = P_{22}(\nu) = \frac{3e^2\omega_H}{4\pi^2 c^2 \xi^2 \sin^2\theta}\left(\frac{\nu}{\nu_c}\right)^2\frac{\psi^2}{\xi^2}\left(1+\frac{\psi^2}{\xi^2}\right)K_{1/3}^2(g_\nu) \tag{3.25}$$

where $g_\nu = g_n$ [see eq.(3.21)].

We should note here that eqs.(3.20) - (3.25) can be extended to the case in which the emitting particle is moving in a plasma. The refractive index will be approximately equal to $n = 1 - \frac{\omega_0^2}{2\omega^2}$, where $\omega_0 = \sqrt{4\pi N_0 e^2/m}$ with $N_0$ being the concentration of the electrons of the plasma. This approximation is correct, provided that $\omega \gg \omega_0$ and $\omega \gg \omega_H^{(0)} = eH/mc$. Under these conditions, in eqs.(3.20) - (3.25), we should note the quantity $\xi$ where it obviously enters as

---

* (cont'd) opposite to that of rotation of the electric vector. This follows from eq.(3.12) when the sign of e and thus of $\omega_H$ is changed. As far as the factor $e\omega_H = e^2H_0/E$ in eq.(3.20) is concerned, it obviously does not depend on the sign of the charge.

$$\eta = \sqrt{\xi^2 + \left(\frac{\omega_0}{\omega}\right)^2} \cdot \qquad (3.26)$$

As is obvious from the preceding, it is also assumed that $\eta \ll 1$.

Equations (3.24), (3.25) and, correspondingly, the Stokes parameters of the emission of a discrete electron differ from those used elsewhere (Ref.1 - 5) by the presence of a factor $\sin^2 \theta$ in the denominator. In this respect, the expressions cited by others (Ref.1 - 5) for the intensity and for the Stokes parameters are incorrect if we are talking about the emission of a discrete electron or a bunch of particles traveling in space.

If, as is often the case, the emission of particles from a volume fixed in space is to be studied, then the expressions given elsewhere (Ref.1 - 5) must be used. Let us now consider this problem.

## 4. Emission of a System of Particles /18

If we are to calculate the total energy flux of emission from eqs.(3.24) and (3.25) for a fixed surface, i.e., calculate the integral of the flux density with respect to all frequencies and directions, this energy will be found to be $1/\sin^2 \theta$ greater than the expression known for the energy losses of an ultrarelativistic particle

$$P = -\frac{dE}{dt} = \frac{2e^4 H_\perp^2}{3m^2c^3}\left(\frac{E}{mc^2}\right)^2 \cdot \qquad (4.1)$$

As pointed out in Section 1, this difference is caused by the nonstationarity of the emission field. Therefore, the total energy flux through a fixed surface

$$P^* = \oint_S \tilde{p}\, ds = P - \frac{\partial}{\partial t}\int_V \frac{E^2 + H^2}{8\pi}\, dV \qquad (4.2)$$

is determined not only by the work P done by the particle but also by the change in field energy on a limited surface S in a volume V. Obviously, the change in field energy is connected with the forward motion of the particle and becomes significant when the potential motion of the particle is equal to the velocity of light.

Actually, this result is due to the deceleration of the exit velocity of propagation of the electromagnetic field. In fact, it would be advisable to study the emission of a discrete electron intersecting an element of volume $r^2 dr d\Omega$ at a distance r from the observer (Fig.5). The electron is in the given element of volume for the time $dt' = dr/v_r$ with $v_r$ the projection of the forward motion of the particles $v_r$ in the direction of the observer. Obviously, $v_r = v_\parallel \cos\theta = v \cos\theta \cos\theta'$. If r(t) is the variable distance to the particle, then the time of observation t is connected with the instant of emission t' by the relation $t = t' + r(t')/c$ (with the emission being considered in a vacuum).

Consequently, the emission of an electron at the time dt′, which corresponds to the migration at a distance dr, will be received by the observer at the time

$$dt = dt'\left(1-\frac{v_\tau}{c}\right) = \left(1-\frac{v_\tau}{c}\right)\frac{dl_\tau}{v_\tau}. \tag{4.3}$$

Hence, it follows that the energy emitted in the time dt′ and passing through unit surface at the point of observation dt is equal to

$$\tilde{p}_\nu dt = \tilde{p}_\nu\left(1-\frac{v_\tau}{c}\right)dt' = p_\nu dt' \tag{4.4}$$

where $\tilde{p}_\nu = \tilde{p}_\nu^{(1)} + \tilde{p}_\nu^{(2)}$ [see eqs.(3.24) and (3.25)] and where the quantity $p_\nu$ is denoted by

$$p_\nu = \tilde{p}_\nu\left(1-\frac{v_\tau}{c}\right) = \tilde{p}_\nu\left(1-\frac{v}{c}\cos\theta\cos\theta'\right). \tag{4.5}$$

As indicated by eq.(4.4), this quantity $p_\nu$ represents the density of the energy flux emitted by the electron in unit time. It can be readily supposed that the integral of $p_\nu$ with respect to all frequencies and directions of the emission leads to the correct expression for the energy losses [i.e., in the ultrarelativistic case, to eq.(4.1)].

Thus, eq.(4.5) determines the connection between the observable emission flux $\tilde{p}_\nu$ and the "power" $p_\nu$ emitted by the electron. Obviously, there exists an analogous relation for all tensor components of the emission polarization [see eqs.(3.15) and (3.23)]:

$$p_{\alpha\beta}(\nu) = \tilde{p}_{\alpha\beta}(\nu)\left(1-\frac{v}{c}\cos\theta\cos\theta'\right). \tag{4.6}$$

/20

In the ultrarelativistic case ($v \simeq c$, $\theta \simeq \theta'$) whence it follows that

$$p_{\alpha\beta}(\nu) = \tilde{p}_{\alpha\beta}(\nu)\sin^2\theta. \tag{4.7}$$

Let us now show that, if we are talking about the emission of particles from a fixed volume, then the quantity $p_{\alpha\beta}(\nu)$ should be used. In fact, this is already clear from eq.(4.4), meaning that eq.(4.4) shows that the energy received by the observer from the trajectory segment dr is determined by the quantity $p_{\alpha\beta}$ and the time $dt' = dr/v_\tau$ during which the electron passes through this segment. Let us examine this problem more closely, with the purpose of obtaining an expression for the intensity and for the other Stokes parameters.

Let us consider the emission intensity of an aggregate of particles whose distribution function is $N(E, \vec{\tau}, \vec{r}, t)$. The quantity $N(E, \vec{\tau}, \vec{r}, t)dEd\Omega_\tau dV$ is found equal to the number of particles having energies in the interval E, E + + dE and velocity directionals within the limits of the solid angle $d\Omega_\tau$; at the instant t, these particles are contained in an element of volume $dV = r^2 dr d\Omega$.

For unit time, the number of particles striking the element of volume is $v_r N(E, \vec{\tau}, \vec{r}, t - \frac{r}{c})\, dE d\Omega_\tau \cdot r^2 d\Omega$ (see Fig.5); here is the instant of observation while $t - \frac{r}{c}$ is the moment of the emission from a fixed point in space. /21
Each particle emits from the volume element an energy equal to $p_\nu dt' = p_\nu \frac{dr}{v_r}$ [see eq.(4.4)]. As a result, the total flux of receivable emission is equal to

$$F_\nu = \int p_\nu \mathcal{N}\left(E, \vec{\tau}, \vec{r}, t - \frac{r}{c}\right) r^2 dr d\Omega dE d\Omega_\tau \,, \tag{4.8}$$

while the emission intensity is equal to

$$I_\nu = \frac{dF}{d\Omega} = \int p_\nu \mathcal{N}\left(E, \vec{\tau}, \vec{r}, t - \frac{r}{c}\right) r^2 dr\, dE\, d\Omega_\tau . \tag{4.9}$$

Obviously, analogous expressions exist for all tensor components

$$I_{\alpha\beta} = \frac{1}{2}\begin{pmatrix} I + Q & U + iV \\ U - iV & I - Q \end{pmatrix} \quad (\alpha, \beta = 1, 2) \tag{4.10}$$

where I, Q, U, V are the Stokes parameters of the receivable emission, namely,

$$I_{\alpha\beta}(\nu) = \int p_{\alpha\beta}(\nu) \mathcal{N}\left(E, \vec{\tau}, \vec{r}, t - \frac{r}{c}\right) r^2 dr\, dE\, d\Omega_\tau . \tag{4.11}$$

For the stationary distribution function, eqs.(4.8) - (4.11) coincide with those given by Ginzburg (Ref.1, 2) at $N(E, \vec{\tau}, \vec{r}, t) \equiv N(E, \vec{\tau}, \vec{r})$.

If we have to do with an aggregate (cloud) of migrating particles, then the observable intensity $I_\nu$ (or the flux $F_\nu$) is essentially determined by the /22
correlation between distribution function and time. In particular, for a discrete electron $N(E, \vec{\tau}, \vec{r}, t - \frac{r}{c}) \sim \delta(r + v_r(t - \frac{r}{c}))$, and as a result of integrating with respect to r in eq.(4.8), we have*

* For simplicity, we will use at this time the velocity averaged with respect to the period of motion, i.e., the velocity $v_\parallel$. In this connection, we will understand by $N(E, \vec{\tau}, \vec{r}, t)$ an average expression for the period due to which the dependence of N on $\vec{\tau}$ is reduced to a dependence only on the angle $\theta$.

$$F_\nu = \frac{P_\nu}{1-(v_z/c)} = \tilde{P}_\nu = \tilde{P}_\nu^{(1)} + \tilde{P}_\nu^{(2)} \qquad (4.12)$$

which should also agree with eqs.(3.24) and (3.25). Let us now assume that we have to do with the stationary "cloud" of particles traveling as a whole with a velocity $\vec{V}$ and a velocity projection in the direction of the observer of $V_r$. This means that in eq.(4.9), the function $N(E, \vec{\tau}, \vec{r}, t) = N_0(E, \vec{\tau}, \vec{r} - \vec{V}t)$. The intensity of emission from such a cloud is equal to $I_\nu = I_\nu^{(0)}\left(1 - \frac{V_r}{c}\right)^{-1}$ where

$$I_\nu^{(1)} = \int P_\nu N_0(E, \vec{\tau}, \vec{r})\, r^2\, dr\, dE\, d\Omega_\tau.$$

Obviously, $I_\nu^{(0)}$ is the intensity of emission of a stationary cloud in which the emitting particles have a concentration of $N_0$.

## 5. Reabsorption of Emission of Ultrarelativistic Particles /23

If a sufficiently large number of emitting particles is in the line of sight, then the absorption and forced (induced) emission of the magnetic-acceleration emission by the relativistic particles themselves begin to manifest themselves. Usually, this process is called reabsorption or resorption. In principle, reabsorption may significantly change the intensity and polarization of the emission. Moreover, negative reabsorption [i.e., emission amplification (Ref.9, 10)] is possible under certain conditions. This means that the system of emitting electrons acts like a maser.

In a study of reabsorption (Ref.9) as well as in earlier studies (Ref.12), expressions averaged with respect to all directions were used for the emission intensity of an individual particle. The applicability of such an approach is ambiguous. It is sufficient to point out that the emission has a finite angular distribution; moreover, its polarized properties depend essentially on the angle $\Psi = \theta - \theta'$ between the direction of the velocity and the direction of the emission [see eqs.(3.24) and (3.25)]. Therefore, when reabsorption and, especially, negative reabsorption are analyzed with consideration of the emission polarization, a stricter analysis of the angular and polarized properties of the magnetobrems emission is necessary.

Below, we will present corresponding results for the magnetobrems emission of ultrarelativistic particles.

The most complete characteristic of the emission is the tensor $I_{\alpha\beta}(\nu)$ [see eq.(4.10)]. Within the limits of applicability of geometric optics, the tensor $I_{\alpha\beta}$ - under stationary conditions - can be found from the equations of transference of the emission, which has the form: /24

$$\frac{dI_{\alpha\beta}}{d\tau} = \varepsilon_{\alpha\beta} - \mathcal{R}_{\alpha\beta\sigma\tau} I_{\sigma\tau}. \tag{5.1}$$

Here,

$$\varepsilon_{\alpha\beta} = \int \nu^2 p_{\alpha\beta} \mathcal{N}(E, \vec{\tau}, \vec{\nu}) dE d\Omega_{\nu} \tag{5.2}$$

where $\varepsilon_{\alpha\beta}$ is the emissivity per unit volume, i.e., the power of the spontaneous emission of unit volume into a discrete solid angle and in an individual frequency interval, while $R_{\alpha\beta\sigma\tau}$ is the reabsorption tensor which allows for absorption and induced emission by the relativistic electrons as well as for absorption and change in polarization in the medium, if such takes place. If the emitting electrons are in a nonrelativistic plasma, we will assume that the index of refraction is $\eta = 1 - \omega_0^2/2\omega^2$ and that the condition $\eta = \sqrt{\xi^2 + (\omega_0/\omega)^2} \ll 1$ is satisfied [see eq.(3.26)].

Calculation of the tensor $R_{\alpha\beta\sigma\tau}$ was carried out elsewhere (Ref.7) for a medium with an index of refraction $\eta = 1$, but under consideration of the Faraday rotation of the plane of polarization. However, that paper contains an incorrect determination of the probability of spontaneous emission per unit volume [and, correspondingly, of the emissivity $\varepsilon_{\alpha\beta}$; see eq.(5.2)]; namely, for its calculation, instead of $p_{\alpha\beta}$, the quantity $\tilde{p}_{\alpha\beta}$ was used (see Sect.4).

The tensor $R_{\alpha\beta\sigma\tau}$ was calculated by Sasonov (Ref.14) for the magnetobrems emission of ultrarelativistic electrons. In the case of an isotropic distribution function* $N(E, \vec{\tau}, \vec{r}) = N(E, \vec{r})$, the tensor components $R_{\alpha\beta\sigma\tau}$, with respect to the axes $\vec{\iota}_1$, $\vec{\iota}_2$ [see eq.(3.4)], are equal to** /25

$$\mathcal{R}_{1111} = -\frac{c^2}{\nu^2} \int \alpha E E^2 \frac{\partial}{\partial E} \left( \frac{\mathcal{N}(E, \vec{\tau})}{E^2} \right) \int \nu^2 p_{11}(\nu) d\Omega$$

$$\mathcal{R}_{2222} = -\frac{c^2}{\nu^2} \int \alpha E E^2 \frac{\partial}{\partial E} \left( \frac{\mathcal{N}(E, \vec{\tau})}{E^2} \right) \int \nu^2 p_{22}(\nu) d\Omega$$

---

* In fact, it is necessary only that the distribution function $N(E, \vec{\tau}, \vec{r})$ change little with the change $\vec{\tau}$ in the directions close to the direction of observation $\vec{k}$. For the concrete case, eqs.(5.3) of $R_{1111}$ and $R_{2222}$ are approximately correct and are also almost correct for the anisotropic distribution function if the condition $\partial N/\partial\theta \ll \eta^{-2} N$ is satisfied. In comparison with $R_{1111}$ and $R_{2222}$, the coefficients $R_{1112} = R_{2111} = \ldots$ etc. have an order of magnitude of $\eta$ [see eqs.(5.4) and (5.5) below]; however, they essentially depend on the degree of anisotropy of the distribution function. Therefore, for the anisotropic distribution function, eqs.(5.3) can be used for calculating these coefficients only if $\partial N/\partial\theta \ll N$. Under these conditions, $N(E, \vec{r})$ in eqs.(5.3) should stand for the function $N(E, \vec{\tau}, \vec{r})$ when $\vec{\tau} = \vec{k}$.

** See Note at the end of this article.

$$\mathcal{R}_{1112} = \mathcal{R}_{2111} = \mathcal{R}_{2212} = \mathcal{R}_{2122} = -\mathcal{R}_{1121} = -\mathcal{R}_{1211} = -\mathcal{R}_{2221} = -\mathcal{R}_{1222} =$$

$$= -\frac{c^2}{2\nu^2}\int dE\, E^2 \frac{\partial}{\partial E}\left(\frac{\mathcal{N}(E,\vec{r})}{E^2}\right)\int r^2 p_{12}(\nu)\, d\Omega$$

$$\mathcal{R}_{1212} = \mathcal{R}_{2121} = \frac{\mathcal{R}_{1111} + \mathcal{R}_{2222}}{2} \qquad (5.3)$$

$$\mathcal{R}_{1122} = \mathcal{R}_{1221} = \mathcal{R}_{2112} = \mathcal{R}_{2211} = 0.$$

The integrals with respect to $d\Omega = 2\pi \sin\theta d\theta \simeq 2\pi \sin\theta d\psi$ contained in eqs.(5.3) are equal to ($\psi = \theta - \theta'$; the fact that the $\psi$ are small makes it possible to assume that $\theta' = \theta$) /26

$$\int r^2 p_{\substack{11\\22}}(\nu)\, d\Omega = \frac{\sqrt{3}}{2}\frac{e^2\omega_H \sin\theta}{\eta}\frac{\nu}{\nu_c}\left[\int_{\nu/\nu_c}^{\infty} K_{5/3}(z)dz \pm K_{1/3}\left(\frac{\nu}{\nu_c}\right)\right] \qquad (5.4)$$

$$\int r^2 p_{12}\, d\Omega = \frac{2i}{\sqrt{3}} c^2 \omega_H \cos\theta\left[\int_{\nu/\nu_c}^{\infty} K_{1/3}(z)dz + \frac{\nu}{\nu_c} K_{1/3}\left(\frac{\nu}{\nu_c}\right)\right],$$

where

$$\nu_c = \frac{3\sin\theta}{4\pi}\cdot\frac{\omega_H}{\eta^3}. \qquad (5.5)$$

It is also assumed that $\theta \gg \eta$, $\theta \gg \pi - \eta$ [when n = 1, obviously $\eta = \xi$, and eq.(5.5) coincides with eq.(3.22)].

For the integration with respect to the energy E in eqs.(5.2) and (5.3), it is important that, for large values $\nu/\nu_c$, the expressions (5.4) are exponentially small. On the other hand,

$$\frac{\nu}{\nu_c(E)} = \frac{\omega_0^2}{3\pi\sin\theta}\frac{mc}{eH\nu}\left(\frac{E_0}{E}\right)^2\left[1+\left(\frac{E}{E_0}\right)^2\right]^{3/2}, \qquad (5.6)$$

where

$$E_0 = mc^2\frac{\omega}{\omega_0} = mc^2\frac{2\pi\nu}{\omega_0}. \qquad (5.7)$$

Therefore, in the integrals (5.2) and (5.3), only the energy region $E_1 < E < E_2$ is important, for which

$$\frac{\nu}{\nu_c(E_1)} \gg 1, \quad \frac{\nu}{\nu_c(E_2)} \gg 1, \quad E_1 \ll E_0 \ll E_2. \qquad (5.8)$$

The behavior of the energy spectrum of the electrons outside the interval (5.8)/27 is only little elucidated by the value of the integrals (5.2) and (5.8). Thus, if the electron spectrum is given in the energy interval $E_1$, $E_2$ and if the conditions (5.8) are satisfied, then the integration with respect to dE can be extended over the entire energy interval.

Let us note that the first of the conditions (5.8) factually does not depend on the presence of a plasma. Actually,

$$\frac{\nu}{\nu_c(E_1)} \simeq \frac{4\pi}{3\sin\theta}\frac{mc\nu}{eH}\left(\frac{mc^2}{E_1}\right)^2 \qquad \text{when } E_1 \ll E_0. \tag{5.9}$$

On the other hand, the second expression in the system (5.8) largely depends on the concentration of the plasma

$$\frac{\nu}{\nu_c(E_2)} \simeq \frac{\omega_0^2}{6\pi^2\sin\theta}\frac{mc}{eH\nu^2}\frac{E_2}{mc^2} \qquad \text{when } E_2 \gg E_0. \tag{5.10}$$

The conditions under which the integration can be extended over the entire energy interval at complete absence of a plasma were given by Ginzburg (Ref.1, 2).

Let us assume that, in the energy interval $E_1 < E < E_2$, the differential energy spectrum of electrons may approximate the function

$$N(E,\vec{r}) = \frac{N_e(\vec{r})}{mc^2}\left(\frac{mc^2}{E}\right)^{\gamma} = \frac{1}{4\pi}K_e E^{-\gamma}. \tag{5.11}$$

Obviously, in the case of isotropy, $KeE^{-\gamma}dE$ is the concentration of particles with energy in the interval E, E + dE. Substituting eq.(5.11) into eqs.(5.2) /28 and (5.3), we obtain

$$\mathcal{R}_{1111}(\gamma) = \frac{\gamma+2}{m\nu^2}\,\varepsilon_{11}(\gamma+1) \tag{5.12}$$

and, analogously,

$$\mathcal{R}_{2222}(\gamma) = \frac{\gamma+2}{m\nu^2}\,\varepsilon_{22}(\gamma+1)$$

$$\mathcal{R}_{1112}(\gamma) = \frac{\gamma+2}{2m\nu^2}\,\varepsilon_{12}(\gamma+1). \tag{5.13}$$

Here, $\varepsilon_{\alpha\beta}(\gamma + 1)$ is the emissivity [eq.(5.2)] calculated for a power-law spectrum (5.11) with an exponent $\gamma + 1$ instead of $\gamma$.

Thus, the components of the reabsorption tensor for an electron system with an exponent of the power-law spectrum $\gamma$ are expressed by the components of the emissivity tensor $\varepsilon_{\alpha\beta}$, which corresponds to the exponent of the spectrum $\gamma + 1$.

The equation of transference (5.1) can be written in a more graphic form by introducing, instead of $I_{\alpha\beta}$, a column vector of Stokes parameters*.

$$S_i = \begin{pmatrix} I \\ Q \\ V \\ U \end{pmatrix} \equiv \begin{pmatrix} I_{11} + I_{22} \\ I_{11} - I_{22} \\ i(I_{21} - I_{12}) \\ I_{12} + I_{21} \end{pmatrix}. \tag{5.14}$$

If it is a question of spectral densities, they can be found with the aid of the index $\nu$ ($I_\nu$, $Q_\nu$, etc.).

With these notations, eq.(5.1) has the form

$$\frac{dS_i}{d\tau} = \varepsilon_i - \mathcal{R}_{ij} S_j \qquad (i, j = 1, 2, 3, 4) \tag{5.15}$$

where $\varepsilon_i$ are the Stokes parameters corresponding to the emissivity tensor /29

$$\varepsilon_i = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{pmatrix} \equiv \begin{pmatrix} \varepsilon_I \\ \varepsilon_Q \\ \varepsilon_V \\ \varepsilon_U = 0 \end{pmatrix} = \begin{pmatrix} \varepsilon_{11} + \varepsilon_{22} \\ \varepsilon_{11} - \varepsilon_{22} \\ i(\varepsilon_{21} - \varepsilon_{12}) \\ \varepsilon_{21} + \varepsilon_{12} \end{pmatrix} \tag{5.16}$$

while the matrix of the reabsorption coefficients $R_{ij}$ is equal to

$$\mathcal{R}_{ij} = \begin{pmatrix} \rho & \lambda & \rho & 0 \\ \lambda & \rho & 0 & 0 \\ \rho & 0 & \rho & 0 \\ 0 & 0 & 0 & \rho \end{pmatrix}, \tag{5.17}$$

where

$$\begin{aligned} \rho(\gamma) &= \frac{1}{2}\left(\mathcal{R}_{1111} + \mathcal{R}_{1122}\right) = \frac{\gamma+2}{2m\nu^2}\varepsilon_I(\gamma+1) \\ \lambda(\gamma) &= \frac{1}{2}\left(\mathcal{R}_{1111} - \mathcal{R}_{1122}\right) = \frac{\gamma+2}{2m\nu^2}\varepsilon_Q(\gamma+1) \\ \rho(\gamma) &= -2i\,\mathcal{R}_{1112} = \frac{\gamma+2}{2m\nu^2}\varepsilon_V(\gamma+1). \end{aligned} \tag{5.18}$$

* The quantities $S_i$ form no vector with respect to the rotation of the axes 1, 2 in the plane of the paper. The law of conversion of $S_i$ is determined by their relation [eq.(4.10)] with the tensor components $I_{\alpha\beta}$.

The fact that the quantity $\mathcal{E}_u$ and certain matrix components $R_{ij}$ are equal to zero is connected with the selection of the system of coordinates. For converting to the last expressions in the system (5.18), eqs.(5.12) and (5.13) were used.

Let us study some of the consequences of the relations obtained.

Of considerable interest is the case in which the magnetic field in the line of sight is more or less uniformly distributed with respect to direction. For such a chaotic field the equation of transference can be averaged with respect to the field orientations. It should be noted that the above-mentioned tensors $\mathcal{E}_{\alpha\beta}$ and $R_{\alpha\beta\sigma\varepsilon}$ were calculated in a special system of coordinates in which the projection of the magnetic field onto the plane of the paper is directed along the axis $\vec{\imath}_2$. To convert to a certain fixed system of coordinates in which the direction of the magnetic field is arbitrary, it is necessary to use the tensor properties of the quantities $\mathcal{E}_{\alpha\beta}$ and $R_{\alpha\beta\sigma\tau}$. /30

As a result of averaging eq.(5.1) with respect to the field directions and as a result of converting to the form of eq.(5.15), it is easy to derive the following relations, with the vinculum denoting the averaging with respect to the field directions:

$$\bar{\mathcal{E}}_Q = \bar{\mathcal{E}}_U = \bar{\mathcal{E}}_V = 0; \qquad \bar{\lambda} = \bar{\rho} = 0 \tag{5.19}$$

$$\bar{\mathcal{E}}_I = \frac{1}{2}\int_0^{\pi} \mathcal{E}_I(\theta)\sin\theta\, d\theta \tag{5.20}$$

$$\bar{\mu}(\gamma) = \frac{(\gamma+2)}{2m\nu^2}\,\overline{\mathcal{E}_I(\gamma+1)}. \tag{5.21}$$

Equations (5.19) indicate that the natural emission is completely depolarized, which should be the case whenever there is no isolated direction. Moreover,

$$\mathcal{E}_i = \mathcal{E}_I\,\delta_{1i}, \quad \mathcal{R}_{ij} = \mu\delta_{ij}, \quad (i = 1,2,3,4). \tag{5.22}$$

If it is a question of the natural emission of a certain finite volume, then eqs.(5.15) with the parameters (5.22) should be solved with the boundary conditions $S_i = 0$ at the distant boundary of the source volume [integration of eq.(5.15) should be carried out along the ray from the far limit to the near limit of the source]. For a homogeneous source, we have the solution: /31

$$I = \frac{\bar{\mathcal{E}}_I}{\bar{\mu}}\left(1 - e^{-\bar{\mu}\ell}\right), \quad Q = U = V = 0, \tag{5.23}$$

where $\ell$ is the extension of the source to the line of sight.

When $\gamma > -2$, the coefficient of reabsorption $\mu$ is always positive, which

corresponds to the absorption [this is clear from eq.(5.21)]. If the optical thickness of the source is great, i.e., $\bar{\mu}l \gg 1$, then $I = \bar{\varepsilon}_1/\mu$ and emission takes place only from a narrow layer of a thickness of the order of $1/\bar{\mu}$. The term $\bar{\varepsilon}_1/\bar{\mu}$ determines the maximum possible intensity when there is positive reabsorption ($\bar{\mu} > 0$).

The necessary condition of negative reabsorption ($\bar{\mu} < 0$) is the condition $\gamma < -2$. However, this condition is not sufficient. For example, in a vacuum, negative reabsorption is generally impossible (Ref.9, 15). This does not contradict eq.(5.21) since, in a vacuum, eq.(5.21) for the power-law spectrum of electrons is valid only if $\gamma > 1/3$ (Ref.2). The conditions for realization of negative reabsorption, for which a plasma must be present, are discussed elsewhere (Ref.9, 10). In the case of negative reabsorption ($\bar{\mu} < 0$) when the optical thickness is $|\mu|l \gg 1$, the intensity of emission is equal to /32

$$I \simeq \frac{\bar{\varepsilon}_1}{\bar{\mu}} e^{|\bar{\mu}|l}$$

and, within the framework of linear theory, can be as large as desired when $l \to \infty$.

Let us emphasize here that the simple form of eq.(5.15) with the coefficients (5.22) is acquired by the equation of transference only in the case of a chaotic field and a rather uniform distribution of the electrons with respect to direction (see footnote to p.15).

In other cases, the equation of transference should be used in its general form (5.1) or (5.15) even if only the variation in I (total emission intensity) is studied. Therefore, the simple concept of one coefficient of reabsorption can be used only in the isotropic case corresponding to the absence of polarization. In this case, by carrying out the necessary calculations of the coefficient $\bar{\mu}$ [see eq.(5.21)], it is obvious that, for electrons in a vacuum (with the refractive index of the medium being $\eta = 1$), we obtain

$$\bar{\mu} = \frac{1}{2}\int_0^{\pi} \mu(\theta)\, d\theta \sin\theta = \bar{g}(\gamma)\frac{e^3}{2\pi m}\left(\frac{3e}{2\pi m^3 c^5}\right)^{\frac{\gamma}{2}} K_e H_\perp^{\frac{\gamma+2}{2}} \nu^{-\frac{\gamma+4}{2}}, \qquad (5.24)$$

where (5.25)

$$\mu(\theta) = g(\gamma)\frac{e^3}{2\pi m}\left(\frac{3e}{2\pi m^3 c^5}\right)^{\frac{\gamma}{2}} K_e H_\perp^{\frac{\gamma+2}{2}} \nu^{-\frac{\gamma+4}{2}} \qquad /33$$

and

$$g(\gamma) = \frac{\sqrt{3}}{4}\Gamma\left(\frac{3\gamma+2}{12}\right)\Gamma\left(\frac{3\gamma+22}{12}\right),$$
$$\bar{g}(\gamma) = \frac{\sqrt{3\pi}}{8}\frac{\Gamma\left(\frac{\gamma+6}{4}\right)}{\Gamma\left(\frac{\gamma+8}{4}\right)}\Gamma\left(\frac{3\gamma+2}{12}\right)\Gamma\left(\frac{3\gamma+22}{12}\right). \qquad (5.26)$$

The accompanying table gives the values of the coefficients $g(\gamma)$ and $\bar{g}(\gamma)$.

| $\gamma$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $g(\gamma)$ | 0.96 | 0.70 | 0.65 | 0.69 | 0.83 |
| $\bar{g}(\gamma)$ | 0.69 | 0.47 | 0.40 | 0.44 | 0.46 |

Equation (5.25) coincides with that used by Ginzburg (Ref.2) and that used in the study of the method of Epstein coefficients for averaged emission. Moreover, the field was considered homogeneous and, in this case, the single quantity $\mu(\theta)$ is insufficient for the characteristics of the emission transfer [see eq.(5.17)].

If the field in the source is homogeneous, then calculation of the polarization of emission is essential for the total intensity. This is so since, generally speaking, all Stokes parameters change; consequently, all components of eqs.(5.15) - (5.17) must be analyzed. However, in the cases of interest here, we can use the fact that $\varepsilon_V/\varepsilon_I \sim \eta \ll 1$ for an angular electron distribution which does change abruptly [see, for example, (Ref.2)] so that $\rho/\mu \sim \eta \ll 1$. Substituting $\varepsilon_V = 0$ and $\rho = 0$ into eqs.(5.15) - (5.17), we obtain a system of two independent equations:

/34

$$\frac{d}{d\tau}\begin{pmatrix} I \\ Q \end{pmatrix} = \begin{pmatrix} \varepsilon_I \\ \varepsilon_Q \end{pmatrix} - \begin{pmatrix} \mu & \lambda \\ \lambda & \mu \end{pmatrix}\begin{pmatrix} I \\ Q \end{pmatrix} \tag{5.27}$$

$$\frac{d}{d\tau}\begin{pmatrix} V \\ u \end{pmatrix} = -\begin{pmatrix} \mu & 0 \\ 0 & \mu \end{pmatrix}\begin{pmatrix} V \\ u \end{pmatrix}. \tag{5.28}$$

If the characteristic radiation of the source is considered, then it can be assumed that, at its far limit, all $S_i = 0$. Integrating eq.(5.27) with this boundary condition, we obtain "at the exit" of the source:

$$I = \frac{\varepsilon_I + \varepsilon_Q}{2(\mu+\lambda)}\left(1 - e^{-(\mu+\lambda)\ell}\right) + \frac{\varepsilon_I - \varepsilon_Q}{2(\mu-\lambda)}\left(1 - e^{-(\mu-\lambda)\ell}\right)$$

$$Q = \frac{\varepsilon_I + \varepsilon_Q}{2(\mu+\lambda)}\left(1 - e^{-(\mu+\lambda)\ell}\right) - \frac{\varepsilon_I - \varepsilon_Q}{2(\mu-\lambda)}\left(1 - e^{-(\mu-\lambda)\ell}\right) \tag{5.29}$$

$$u = 0, \qquad V = 0.$$

Equations (5.29) have a simple meaning. If, instead of the Stokes parameters I and Q, we use the parameters $I^{(1)} = I_{11} = \frac{1}{2}(I + Q)$ and $I^{(2)} = I_{22} = \frac{1}{2}(I - Q)$ as

well as the analogous parameters for the emissivity $\varepsilon^{(1)} = \frac{1}{2}(\varepsilon_I + \varepsilon_Q)$, $\varepsilon^{(2)} = \frac{1}{2}(\varepsilon_I - \varepsilon_Q)$, then eq.(5.28) is reduced to

$$I^{(1)} = \frac{\varepsilon^{(1)}}{\mu + \lambda}\left[1 - e^{-(\mu+\lambda)\ell}\right],$$
$$I^{(2)} = \frac{\varepsilon^{(2)}}{\mu - \lambda}\left[1 - e^{-(\mu-\lambda)\ell}\right]. \qquad (5.30)$$

It follows from this that the emission intensities with two main polarizations /35 behave independently. Moreover, for the intensity $I^{(1)}$, which corresponds to the polarization across the projection of the field onto the plane of the paper, the coefficient of reabsorption is equal to $\mu_\perp = \mu + \lambda$ while, for an emission with polarization along the projection of the field, the intensity $I^{(2)}$ is the coefficient of reabsorption $\mu_\parallel = \mu - \lambda$. For a thin layer at $|\mu \pm \lambda|\ell \ll 1$, we have

$$\frac{I^{(1)}}{I^{(2)}} \equiv \frac{I_0^{(1)}}{I_0^{(2)}} = \frac{\varepsilon^{(1)}}{\varepsilon^{(2)}}. \qquad (5.31)$$

If the reabsorption is positive, i.e., $\mu + \lambda > \mu - \lambda > 0$ (or the equivalent $\mu > \lambda > 0$), then, if the optical thickness is sufficiently large $(\mu - \lambda)\ell \gg 1$, we obtain from eq.(5.30):

$$1 \leq \frac{I^{(1)}}{I^{(2)}} = \frac{\varepsilon^{(1)}}{\varepsilon^{(2)}}\frac{\mu-\lambda}{\mu+\lambda} \leq \frac{\varepsilon^{(1)}}{\varepsilon^{(2)}} \equiv \frac{I_0^{(1)}}{I_0^{(2)}}. \qquad (5.32)$$

This inequality shows that positive reabsorption always leads to a depolarization of the emission (when $v = u = 0$, the polarization of emission is

$$\Pi = \frac{1}{I}\sqrt{Q^2 + V^2 + U^2} = \frac{I^{(1)} - I^{(2)}}{I^{(1)} + I^{(2)}}).$$

This result is completely understandable since the emissivity $\varepsilon^{(1)}$ (polarization across the field) is greater than $\varepsilon^{(2)}$ for magnetobrems emission; this means that the reverse process (namely, absorption) is stronger for waves with polariza- /36 tion (electric vector) directed across the field. The picture is different for negative reabsorption. Thus, if $\mu + \nu < 0$ or $\mu - \nu < 0$, then, if the quantities $\ell$ are sufficiently large, it follows from eqs.(5.29) that

$$|Q| = \left|I^{(1)} - I^{(2)}\right| \to I = I^{(1)} + I^{(2)}, \quad \Pi \to 1 \qquad (5.33)$$

i.e., the completely linear polarized emission is perpendicular to the projection of the magnetic field onto the plane of the paper. This is a result of the greater intensification of the polarization predominating in spontaneous emission. The strong linear polarization ($\geq 75\%$) serves as an indicator of the important role played by negative reabsorption for the given source.

With negative reabsorption, the dependence of the coefficients (5.18) on the angle between the line of sight and the magnetic field may also be important. If the field in the source is nonuniform (but not completely chaotic), there will be an intensification of emission chiefly in those directions where $\mu$ is maximum, i.e., in directions across the field. Therefore, when the thickness is great, the individual regions of the heterogeneous source will seem abnormally bright.

In other words, the spatial intensity fluctuations at the source will increase sharply.

Finally, let us study the possibility of the appearance of notable elliptical or circular polarization. If we disregard the anisotropic electron distribution with respect to velocity as well as the important role of the Faraday rotation in the source, then elliptical polarization with an accuracy to terms of the order of $\eta$ will be absent. /37

Under cosmic conditions, there is little probability for the long-time existence of sharply anisotropic electron velocity distributions, as was the case for a particle bundle. However, this might occur as a sporadic event and may also be quasi-stationary in a relatively strong magnetic field (with a moderately dense bundle of particles moving along the magnetic field). It is even easier to realize conditions in which the Faraday rotation of the polarization plane must be considered or, more accurately, conditions in which the plasma must be assumed as magnetoactive (above, we did not assume the plasma as being isotropic, for which reason we used one refractive index $\eta$). As is known, the propagation and emission of waves in a magnetoactive plasma differ by the great diversity of the various possibilities [see (Ref.18) and the literature cited there].

Further studies should be made on the analysis of positive and negative reabsorptions of magnetobrems emission for the case of sharply anisotropic distribution functions of the emitting electrons with respect to velocity and for the case of inhomogeneity (magnetoactivity) of the relatively cold "parent" plasma in which the emitting relativistic particles are moving.

## 6. Variations in the Magnetic Field due to Magnetobrems Emission (Losses) /38

In studying the emission of a charged particle moving in a magnetic field as well as in studying the energy losses or gains by the particle due to other mechanisms, the magnetic field is usually regarded as fixed. It is obvious that such a statement of the problem has a limited range of applicability. Actually, a particle traveling in a magnetic field creates its own magnetic field $\vec{H}_1$ which weakens the external field $\vec{H}_0$ (diamagnetic effect). The field $\vec{H}_1$ depends on the energy of the particle E and decreases when there is a decrease in this energy $E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$. Therefore, considering the energy losses, the field $\vec{H}_1$ decreases, which may lead to a change not only in the total field $\vec{H} = \vec{H}_0 + \vec{H}_1$ but

also in the field $\vec{H}_0$ (we are talking here about mutual inductance; see below). As a result of the change in the magnetic field, an induced electric field $\vec{\mathcal{E}}$ emerges which, in its turn, may change the energy of the particle. Recently, the question was raised as to whether a particle may "extract" energy from the field and thereby not only lose its own kinetic energy $E_k = E - mc^2$ but also a large amount of general energy (Ref.16). As will be shown later, this conclusion is erroneous; nevertheless, the energy relationships, along with a study of energy loss (or gain) during travel of the particle in the magnetic field are of interest; for this reason, we will investigate this further. /39

When a particle moves in a homogeneous magnetic field with an inhomogeneity* $\vec{H}_0$, then the particle with a charge e and a mass m will have the magnetic moment

$$\vec{\mu} = -\frac{m v_\perp^2 \vec{H}_0}{2 H_0^2 \sqrt{1 - v^2/c^2}} = -\frac{|e| r_H v_\perp}{2c}\left(\frac{\vec{H}}{H_0}\right) = -\frac{v_\perp^2 E}{2c^2 H_0}\left(\frac{\vec{H}}{H_0}\right). \qquad (6.1)$$

In fact, the cyclotron frequency of the particle in the magnetic field is $\omega_H = \frac{|e|H_0}{mc}\frac{mc^2}{E} = \frac{|e|H_0}{mc}\sqrt{1 - v^2/c^2}$ while the radius of the projection of the orbit onto a surface perpendicular to $\vec{H}_0$, is $r_H = \frac{v_\perp}{\omega_H} = \frac{mv_\perp}{|e|H_0\sqrt{1 - v^2/c^2}}$. Finally, the magnetic moment is $\vec{\mu} = \frac{e}{2c}[\vec{r}\,\vec{v}]$ which leads to eq.(6.1) where $v_\perp$ is the projection of the velocity $v_\perp$ onto the surface perpendicular to the field $\vec{H}_0$. The sign in eq.(6.1) can be selected from general considerations, since it is known that the gas of the charged particles is diamagnetic (disregarding the spin). If there are numerous particles and if they travel independently, then their moments are simply summed up. Moreover, the eigenfield of all particles is small in comparison with the external field $\vec{H}_0$ (this field is created by sources placed outside the region under study) when $4\pi N\mu \ll H_0$, where $\mu = |\vec{\mu}|$ and N represents the concentration of particles (moments). More exactly, if /40 we are talking about particles with different values for $\mu$, then $N_\mu$ acts as the total moment per unit volume, i.e., as the magnetization M. The inequality $4\pi M \ll H_0$ in terms of the theory of magnetics clearly indicates that $B = H_0 + 4\pi M \simeq H_0$. (Hence, the appearance of the factor $4\pi$ in the given inequality is also obvious.)

Considering eq.(6.1), we thus arrive at the condition of the weakness of the diamagnetic effect in the form

$$\frac{\overline{v_\perp^2 E}\,N}{4c^2} \ll \frac{H_0^2}{8\pi}, \qquad (6.2)$$

* We are speaking here of the magnetic induction $\vec{B}_0$. However, with the exception of the last portion of this Section, we are concerned with the case of a vacuum, for which reason we will not - if possible - distinguish between $\vec{H}$ and $\vec{B}$.

where the vinculum denotes averaging with respect to the energy spectrum. For isotropic distribution of the ultrarelativistic particles (specifically, we are thinking here of cosmic rays) with respect to the velocity directions, we have $\frac{\overline{v_\perp}^2}{c^2} = \frac{2}{3}$ and the condition (6.2) can be written in the form

$$W_{c.r} \ll 6 W_M, \quad W_{c.r} = \bar{E} N, \quad W_M = \frac{H_0^2}{8\pi}, \quad E \gg mc^2, \tag{6.3}$$

where the value of the particle mass m plays no role.

Thus, in order for the effect of the relativistic particles on the magnetic field to be small, this energy density must be small in comparison with the density of the magnetic energy. Meanwhile, under cosmic conditions, we have the following in a series of cases (Ref.1, 17):

$$W_{c.r} \sim W_m. \tag{6.4}$$

Under such conditions, the relativistic particles obviously affect the field /40a
but, generally speaking, the field may still remain strong (in the sense that the field is located in a medium of the order of the external field $H_0$). If

$$W_{c.r} \gg W_M = \frac{H_0^2}{8\pi}, \tag{6.5}$$

then the dynamic effect should lead to a complete screening of the field, to instabilities, etc. Further development of such considerations makes it possible to obtain additional arguments in favor of the relation (6.4) or, more exactly, of the relation $W_{c.r} \lesssim W_H$ against the possibility of realizing the condition /41
[eq.(6.5)]. Without further considering this interesting problem [see elsewhere (Ref.17)], we will now study another case of a single particle whose properties and state are written in the form e, m, $E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$, $v_\perp$; we will assume the external field $\vec{H}_0$ as homogeneous and as generated in a rather long solenoid (Fig.6). The current flowing in the "winding" of the solenoid refers to unit length of the solenoid, $i = \int j de = \frac{c}{4\pi} H_0$, where j is the current density in the "winding" (disregarding the shielding i = jd, where d is the thickness of the "winding"). Let us assume that the particle trajectory is located entirely within the solenoid and that it is situated at a relatively great distance from its walls. The volume of the solenoid is $V = \pi r_0^2 L$, $L \gg r_0$.

The equation of motion of the particle has the form

$$\frac{d}{dt} \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} = e\left\{\vec{E} + \frac{1}{c}[\vec{v}\,\vec{H}]\right\} - \vec{f}, \tag{6.6}$$

where $\vec{f}$ is a certain "friction force" leading to energy losses. After multiplication by the velocity $\vec{v}$, we obtain

$$\frac{dE}{dt} = e\vec{\mathcal{E}}\vec{v} - P, \quad E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}, \quad P = \vec{f}\vec{v}; \tag{6.7}$$

If there is acceleration and no losses, then $P < 0$; the force of radiative friction obviously is included in the expression for $\vec{f}$.

Let us designate the field-generating current density by $\vec{j}$. This does not include the current connected with the given particle; its density is equal to $e\vec{v}\delta(\vec{r} - \vec{r}_e)(t)$. Then, the Poynting theorem derived from the field equations, should be written in the form /42

$$\frac{\partial}{\partial t}\left\{\frac{\mathcal{E}^2 + H^2}{8\pi}\right\} + e\vec{\mathcal{E}}\vec{v}\delta(\vec{r} - \vec{r}_e) = -\frac{c}{4\pi}\operatorname{div}[\vec{\mathcal{E}}\vec{H}] - \vec{j}\vec{\mathcal{E}} \tag{6.8}$$

or, after integrating with respect to a given volume V and considering eq.(6.7), in the form

$$\frac{d}{dt}\left\{\int \frac{\mathcal{E}^2 + H^2}{8\pi}\, dv + E\right\} = -\oint \vec{S}\vec{n}\, d\sigma - \int \vec{j}\vec{\mathcal{E}}\, dv - P, \tag{6.9}$$

where the Poynting vector $\vec{S} = \frac{c}{4\pi}[\vec{\mathcal{E}}\,\vec{H}]$ is integrated with respect to the surface bounding the volume V (obviously, $\vec{n}$ is the external normal to this surface).

Equations (6.6) - (6.9) understandably have only a general character, but we will use them for the case of the field located in a solenoid (in the absence of a particle, the field $\vec{H} = \vec{H}_0 = \text{const}$). Inside the solenoid, the total field is $\vec{H} = \vec{H}_0 + \vec{H}_1$ where $\vec{H}_1$ is the field created by the particle itself. For simplicity, we will consider it as moving in a circle. At a relatively great distance $r \gg r_H$ from the particle trajectory, its field averaged over the period is equivalent to the field of the magnetic moment (6.1) with $v_\perp = v$. Therefore, far from the particle

/43

$$\vec{H}_1 = \operatorname{curl}\operatorname{curl}\frac{\vec{\mu}}{r}, \quad \vec{\mathcal{E}}_1 = -\frac{1}{c}\operatorname{curl}\frac{\vec{\mu}}{r}, \tag{6.10}$$

where the value $\vec{\mu}(t')$ should be taken at the instant $t' = t - r/c$ [see (Ref.12, Sect.72)].

Let us now assume that the winding of the solenoid is at a distance r from the particle, which is much smaller than the "wavelength" $\lambda = \frac{c}{\tau}$, where $\tau$ is

the characteristic time of change in momenta due to the losses $\left(\frac{d\mu}{dt} \equiv \dot{\mu} \sim \mu/\tau\right)$.

In this case, i.e., disregarding the delay, we have $\vec{\mu}(t') = \vec{\mu}(t)$ and

$$\vec{H}_1 = \operatorname{curl} \vec{A}_1 = \frac{3\vec{r}(\vec{r}\vec{\mu}) - \vec{\mu} r^2}{r^5}, \quad \vec{A}_1 = \left[\nabla \frac{1}{r}, \vec{\mu}\right], \quad \vec{\mathcal{E}}_1 = -\frac{1}{c}\dot{\vec{A}} =$$

$$= -\frac{1}{c}\left[\nabla \frac{1}{r}, \dot{\vec{\mu}}\right] = -\frac{1}{c}\left[\dot{\vec{\mu}}, \frac{\vec{r}}{r^3}\right]. \tag{6.11}$$

Let us now make use of eq.(6.9) by selecting, as the surface of integration, the internal surface of the cylindrical "winding". The fields $H_1$ and $\mathcal{E}_1$ are small quantities with respect to $H_0$ and thus can be written as

$$\int \frac{\mathcal{E}^2 + H^2}{8\pi} dV \simeq \frac{H_0^2}{8\pi} V + \frac{\vec{H}_0}{4\pi}\int \vec{H}_1 dv,$$

$$\oint \vec{S}\vec{n}\, d\sigma \simeq \frac{c}{4\pi}\int \left[\vec{\mathcal{E}}_1 \vec{H}_0\right]_n d\sigma. \tag{6.12}$$

In view of eq.(6.11) and considering that, in the given case, $\vec{n}\vec{\mu} = \vec{n}\dot{\vec{\mu}} = 0$, we have /44

$$\vec{S} \simeq \frac{c}{4\pi}\left[\vec{\mathcal{E}}\vec{H}_0\right] = \frac{1}{4\pi r}\left\{\dot{\vec{\mu}}(\vec{H}_0\vec{r}) - \vec{r}(\dot{\vec{\mu}}\vec{H}_0)\right\}, \quad \oint \vec{S}\vec{n}\, d\sigma = -\frac{\dot{\vec{\mu}}\vec{H}_0}{4\pi}\oint \frac{\vec{r}\vec{n}}{r^3} d\sigma = -\dot{\vec{\mu}}\vec{H}_0. \tag{6.13}$$

Since

$$\oint \frac{\vec{r}\vec{n}}{r^3} d\sigma = -\oint \vec{n}\nabla \frac{1}{r} d\sigma = -\int \Delta \frac{1}{r} dv = 4\pi \int \delta(\vec{r}) dV = 4\pi.$$

Moreover, because of eq.(6.11),

$$\frac{\vec{H}_0}{4\pi}\int \vec{H}_1 dv = \frac{\vec{H}_0}{4\pi}\int \operatorname{curl} \vec{A}_1 dv = \frac{\vec{H}_0}{4\pi}\oint \left[\vec{n}\left[\nabla \frac{1}{r}, \vec{\mu}\right]\right] d\sigma = \frac{\vec{\mu}\vec{H}_0}{4\pi}\oint \frac{\vec{r}\vec{n}}{r^3} d\sigma = \vec{\mu}\vec{H}_0. \tag{6.14}$$

Let us now assume that the field $\vec{H}_0$ is kept constant despite the change in momentum of the particle $\vec{\mu}$ caused by the losses. This can be done only because of the work of external emf sources ("batteries") inserted into the circuit of the winding. Under similar conditions, using eqs.(6.12) - (6.14) and assuming $\vec{H}_0$ = const, eq.(6.9) will take the form*

$$\frac{dE}{dt} = -P(E), \qquad E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \tag{6.15}$$

---

* For footnote see following page.

i.e., we obtain the equation generally used for the energy of a particle in the presence of losses.

Certainly, this result is immediately obvious from eq.(6.7) since, at $\vec{H}_0$ = = const, the electric field is $\vec{\mathcal{E}} = 0$. However, such an examination demonstrates that this originates with a magnetic field and magnetic energy. This field energy, in the volume V (in the solenoid), according to eqs.(6.1), (6.12), and (6.14), is equal to /45

$$\int \frac{H^2}{8\pi} dv \simeq \frac{H_0^2}{8\pi} V + \frac{\vec{H}_0}{4\pi} \int \vec{H}_1 dv = \frac{H_0^2}{8\pi} V + \vec{\mu} \vec{H}_0 =$$

$$= \frac{H_0^2}{8\pi} V - \frac{v^2 E}{2c^2} \quad , \tag{6.16}$$

where we put $v = v_\perp$ (motion in a circle). As the particle loses energy, the momentum $\mu = |\vec{\mu}|$ decreases and the total magnetic energy increases, since $\vec{\mu} \vec{H}_0 < 0$. This increase is due to the energy flux within the solenoid. At the end of the process (the particle has lost energy and its momentum is $\mu = 0$), the field $\vec{H}_0$ presumably remains unchanged and the "batteries" have lost an energy of

$$-\vec{\mu}(0) \vec{H}_0 = \frac{v^2(0) E(0)}{2c^2} \quad , \tag{6.17}$$

where the argument t = 0 indicates the initial values $\vec{\mu}$, v, and E. Another more interesting statement of the problem is the case when the field $\vec{H}_0$ is not considered fixed (Ref.16) with the "winding" of the solenoid closed and the field formed by a flux of electrons that do not encounter resistance (i.e., the electrons describe a circle of radius $r_0$, filling a certain thin layer of thickness d; see Fig.6). Since the conductivity in cosmic plasma is very high, this case has some features that approximate real conditions. However, the degree of /46 this approximation should not be overestimated since, under cosmic conditions, it would be necessary to consider the field as passing through the entire medium

---

* The surface of integration is selected such that the sources of the field $\vec{H}_0$ will be exterior to it because of which $\int \vec{j}\vec{e}\, dv = 0$ [see eq.(6.9)]. To explain conditions under which we can disregard the energy $\int H_1^2/8\pi \cdot dv$, let us make the corresponding evaluation. It is easy to see that $\int \frac{\bar{H}_1^2}{8\pi} dv \sim \frac{\mu^2}{r_H} \sim \mu H_1$ where $\bar{H}_1 \sim \mu/r_H^3$ is a certain average field of momentum $\vec{\mu}$ in the particle orbit. Obviously, the condition $\frac{\vec{H}_0}{4\pi} \int \vec{H}_1 dv = \vec{\mu} \vec{H}_0 \gg \int \frac{H_1^2}{8\pi} dv$ takes the form

$$\bar{H}_1 \ll H_0 \quad \text{or} \quad H_0 \ll \frac{vE^2}{e^3 c} \leqslant \frac{m^2 c^3 v}{e^3} \sim 10^{16} \frac{v}{c}.$$

and within the solenoid. Moreover, for simplicity, we will assume that the "winding" does not distort the field of the particle, i.e., the field of the momenta $\vec{\mu}$. This means that the "winding" should be sufficiently thin ($d \ll \delta$, where $\delta = \sqrt{\frac{mc^2}{4\pi e^2 N_0}}$ is the depth of penetration of the field into the "winding"*. Under these conditions, let us place the surface bounding the volume analyzed in eq.(6.9) outside the winding. Here $H_0 = 0$, and in approximation of eq.(6.12) the energy flux is $S = 0$. Then, eq.(6.14) remains practically unchanged when the shielding is disregarded. Finally, eq.(6.9) takes the form

$$\frac{d}{dt}\left\{\frac{H_0^2}{8\pi} V + \vec{\mu}\vec{H}_0 + E\right\} = -P - \int \vec{j}\vec{\mathcal{E}}\, dv \,, \qquad (6.18)$$

where $\int \vec{j}\vec{\mathcal{E}} dv$ is taken with respect to the volume of the "winding". It is obvious that $\int \vec{j}\vec{\mathcal{E}} dv = \frac{d}{dt}\mathcal{K}$ where $\mathcal{K} = \frac{mu^2}{2} \cdot 2\pi r_0 dLN_0$ is the kinetic energy of the well-regulated electron motion in the winding, responsible for creating the field $H_0$ ($2\pi r_0 dL$ is the volume of the winding and $N_0$ is the concentration of electrons considered as nonrelativistic). As stated before, the current density is $\frac{i}{d} = \frac{cH_0}{4\pi d}$ and, on the other hand, $j = eN_0 u$. Hence, $u = \frac{cH_0}{4\pi e N_0 d}$ and

/47

$$\mathcal{K} = \frac{mc^2}{e^2 N_0 (2\pi r_0 d)} \cdot \frac{H_0^2}{8\pi} V, \quad V = \pi r_0 L \,. \qquad (6.19)$$

Because of the increase in the solenoid radius $r_0$, the factor $\frac{mc^2}{e^2 N_0 (2\pi r_0 d)} = \frac{2\delta^2}{r_0 d}$ can be made considerably less than unity even if we observe the above-mentioned conditions $\delta = \sqrt{\frac{mc^2}{4\pi e^2 N_0}} \ll d$. Under similar conditions, i.e., when $\frac{2\delta^2}{r_0 d} \ll 1$ to which we are limiting ourselves, a calculation of the term $\int \vec{j}\vec{\mathcal{E}} dv$ in eq.(6.18) would only mean the introduction of a minor correction to the term $\frac{d}{dt}\left(\frac{H_0^2}{8\pi} V\right)$. As a result, we arrive at the equation

$$\frac{d}{dt}\left\{\frac{H_0^2}{8\pi} V + \vec{\mu}\vec{H}_0 + E\right\} = -P \,, \qquad (6.20)$$

---

* For a free electron gas, $\varepsilon = 1 - 4\pi e^2 N_0/m\omega^2$; at $\varepsilon < 0$, $|\varepsilon| \gg 1$, the field dampens according to the law

$$e^{-\frac{\omega}{c}\sqrt{|\varepsilon|}\, x} = e^{-x/\delta}, \qquad \delta^2 = \frac{mc^2}{4\pi e^2 N_0} \,.$$

which was applied without comments by Manley (Ref.16). Having expressed the field $\vec{\mathcal{E}}$ by $\frac{d\vec{H}_0}{dt}$, eq.(6.20) must be supplemented by eq.(6.7) for solving the problem. This is easy to do since the field $\vec{\mathcal{E}}$ is a simply induced field so that $2\pi r\,\mathcal{E} = -\frac{1}{c}\pi r^2 \frac{dH_0}{dt}$ or $\mathcal{E} = \frac{r_H}{2e}\frac{dH_0}{dt}$ along the trajectory of the particle, where $r = r_H = \frac{mv_\perp c}{|e|H_0\sqrt{1 - v^2/c^2}}$. Finally,

$$\vec{\mathcal{E}} = \frac{m\vec{v}_\perp \frac{dH_0}{dt}}{2e\sqrt{1-v^2/c^2}\,H_0}, \qquad (6.21)$$

where the sign of $\vec{\mathcal{E}}$ is determined by Lenz' law (to maintain the flow through the contour when $\frac{dH_0}{dt} > 0$, the force eE increases the velocity $v_\perp$). On substituting this field $\vec{\mathcal{E}}$ into eq.(6.7), we obtain /48

$$\frac{dE}{dt} = \frac{mv^2}{2\sqrt{1-v^2/c^2}}\frac{d}{dt}\ln H_0 - P, \qquad (6.22)$$

where we set $v_\perp = v$ since we are considering only the motion of a particle along a circle.

Subtracting eq.(6.23) from eq.(6.20), we have

$$\frac{d}{dt}\left\{W - F\right\} = -\frac{1}{2}F\frac{d}{dt}\ln W, \quad W = \frac{H_0^2 U}{8\pi},$$
$$F = \frac{mv^2}{2\sqrt{1-v^2/c^2}} = \frac{v^2 E}{2c^2}. \qquad (6.23)$$

The integral of this equation is

$$F = 2W_0\left\{\frac{W}{W_0} - \left(1 - \frac{F_0}{2W_0}\right)\sqrt{\frac{W}{W_0}}\right\}, \qquad (6.24)$$

where $W_0$ and $F_0$ are the values of W and F when t = 0.

After the particle has lost energy, $F = F(\infty) = 0$ and, according to eq.(6.24), $W(\infty) = W(0) \simeq -F_0$ for $F_0/W_0 \ll 1$, i.e., the energy change of the external field will be

$$\frac{H_0^2(0)}{8\pi}V - \frac{H_0^2(\infty)}{8\pi}V = \frac{v^2(0)E(0)}{2c^2} = -\mu(0)H_0(0). \qquad (6.25)$$

Integrating eq.(6.20) which can be written in the form $\frac{d}{dt}(W - F + E) = -P$ and considering the relation (6.25), it is easy to demonstrate that the total losses are $\int_0^\infty P dt = E(0)$ since $E(\infty) = F(\infty) = 0$. In other words, the particle in this case loses only an energy $E(0)$; this, naturally, follows from eqs.(6.22), (6.24), and (6.25) if the calculation is made only with an accuracy to terms of the order of $F_0$ but not $\frac{F_0^2}{W_0}$ [this accuracy was assumed in eq.(6.25)]. /49

The significance of the relation (6.25), where the equality $\frac{v^2(0)E(0)}{2c^2} = -\mu(0)H_0(0)$ follows from eq.(6.1) at $v_\perp = v$, is obvious if we remember what was said in connection with eqs.(6.16) and (6.17). The total energy of the magnetic field in the solenoid is

$$\int \frac{H^2}{8\pi} dv \simeq \frac{H_0^2}{8\pi} U + \mu H_0. \qquad (6.16)$$

Moreover, if we assume that $\mu(\infty)H_0(\infty) = F(\infty) = 0$, then eq.(6.25) is simply the condition of conservation of the total magnetic energy. However, in addition, the field $\vec{H} = \vec{H}_0 + \vec{H}_1$ itself changes and becomes redistributed: When the absolute value of the momentum $\mu$ decreases, the field $H_1$ also decreases; therefore, to conserve the total magnetic energy, the homogeneous field $H_0$ should also decrease, since the field $\vec{H}_1$ is directed opposite the field $\vec{H}_0$ (diamagnetic effect).

Thus, the solution finally seems rather trivial: Everything reduces to a consideration of the diamagnetic effect generated when the charged particles travel in the electric field and to an application of the law of conservation of energy (Poynting theorems). Moreover, in both cases studied (constant field $\vec{H}_0$ and solenoid with "short-circuited winding") the particle loses only its energy and cannot "extract" energy from the magnetic field. /50

Remembering that a generalization is possible for the more complicated case of an aggregate of emitting particles, let us examine the problem in this Section by macroscopic means. For this, let us introduce, in addition to the field strength, the magnetic moment of unit volume $\vec{M}$ and the magnetic induction $\vec{B} = \vec{H} + 4\pi\vec{M}$.

If N is the concentration of particles, then the magnetic moment of unit volume is $\vec{M} = N\vec{\mu}$, where we write the magnetic element of a single particle [eq.(6.1)] in the form

$$\vec{\mu} = -\frac{p_\perp^2}{2B}\frac{c^2}{E}\frac{\vec{B}}{B}, \quad E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}, \quad p_\perp = \frac{m v_\perp}{\sqrt{1 - v^2/c^2}}. \qquad (6.1a)$$

The difference between this expression and eq.(6.1) consists only in changing the field $\vec{H}_0$ to the average macroscopic field, i.e., to the induction $\vec{B}$ (for simplicity, we will not differentiate between $\vec{B}$ and $\vec{B}_0$). Let us then calculate the constant of the quantity $K = p_\perp^2/B$ (adiabatic invariant) when there are slow changes in the field, and let us consider only the case of ultrarelativistic particles ($E \gg mc^2$) effecting a circular motion when $E^2 \simeq c^2 p_\perp^2 = Kc^2 B$ and

$$\mu = |\vec{\mu}| = \frac{E}{2B} = \frac{1}{2}\sqrt{\frac{Kc^2}{B}}. \tag{6.26}$$

Let us compute the energy of the field and of the particle system, assuming that the external field as well as the distribution of particles N in the given volume are homogeneous. Then, /51

$$du = \frac{1}{4\pi}\vec{H}d\vec{B} = \frac{1}{4\pi}(\vec{B} - 4\pi\vec{M})d\vec{B} = d\left(\frac{B^2}{8\pi}\right) - \vec{M}d\vec{B}. \tag{6.27}$$

To compute the last term in eq.(6.27), we will use the expression $\vec{M} = N\vec{\mu} = \frac{1}{2} N\sqrt{\frac{Kc^2}{B}} \cdot \frac{\vec{B}}{B}$ whence $\vec{M}d\vec{B} = -d(N\sqrt{Kc^2 B}) = d(2\vec{M}\vec{B})$. As a result, we obtain

$$\mathcal{U} = \frac{B^2}{8\pi} - 2\vec{M}\vec{B}. \tag{6.28}$$

The energy of the particles in unit volume, because of eq.(6.26), is equal to $NE = 2N\mu B = 2\vec{M}\vec{B}$. Thus, the energy density [eq.(6.28)] is composed simply of the energy density of the "true" average field $\frac{B^2}{8\pi}$ and the energy density of the particles NE.

With the aid of the relation $\vec{B} = \vec{H} + 4\pi\vec{M}$, eq.(6.28) can also be written in the form

$$\mathcal{U} = \frac{H^2}{8\pi} - \vec{M}\vec{H} - 6\pi M^2. \tag{6.29}$$

The last term in eq.(6.29) is proportional to the density of the magnetic energy of the currents connected with the particles; when $4\pi M \ll H$ or $NE \ll \frac{H^2}{4}$ [see eq.(6.3)], it can be omitted. Thus, in approximation of the slight magnetization which was used by Manley (Ref.16) and above, we obtain

$$\left.\begin{aligned} \mathcal{U} &= \frac{H^2}{8\pi} - \vec{M}\vec{H} = \frac{H^2}{8\pi} + \vec{M}\vec{H} + \mathcal{N}E \\ \mathcal{N}E &= -2\vec{M}\vec{H} \end{aligned}\right\}. \tag{6.30}$$

Specifically, in the form $U = \frac{H^2}{8\pi} + \vec{M}\vec{H} + NE$ the energy density of the field- /52
particle system is given in brackets and in eq.(6.20).

Let us now consider the change in energy of the system, as a result of the deceleration of particles due to emission or other losses under two slightly different conditions.

Let the external field be kept constant (first statement of the problem). This can be effected, for example, if the layer of the relativistic plasma is confined by the external field (see Fig.7). In this case, we have $\vec{H}_e = \vec{H}_i = \vec{H}$ because of the continuity of the tangential components $\vec{H}$ where the subscripts e and i refer to the values on the external and frictional side of the boundary of the partition.

In the initial state, the energy of the system is equal to $U(0) = \frac{H^2(0)}{8\pi} - \vec{M}(0)\vec{H}(0)$. As a result of the losses, let the kinetic energy of the particles in the final state be equal to zero and, correspondingly, $\vec{M}(\infty) = 0$. Then, in the final state we have $U(\infty) = \frac{H^2(\infty)}{8\pi}$ and the change in energy is

$$\Delta U = U(\infty) - U(0) = \vec{M}(0)\vec{H}(0) = -\frac{1}{2}NE(0). \qquad (6.31)$$

In this case, the total change in energy of the system consists of the energy $NE(0)$ lost by the particles and of the work of the external currents $\frac{1}{2}NE(0) = -\vec{M}(0)\vec{H}(0)$ which maintain constancy of the field in the given volume.

Let us now consider the second case when the system is connected into a superconducting solenoid, so that the electric field will be $\vec{\mathcal{E}} = 0$ at the boundaries of the volume. This case corresponds more closely to the problem /53
studied by Manley (Ref.16) and by us for the short-circuited "winding" of the solenoid. Under these conditions, the total magnetic flux $\int B_{\parallel} d\delta = \text{const}$; therefore, if the cross section is constant and the field is homogeneous, we have $B = \text{const}$.

Consequently, because of eq.(6.28), the total change in energy of the system, after the particles have lost their energy, is equal to [obviously, $\vec{M}(\infty) = 0$]

$$U(\infty) - U(0) = 2M(0)\vec{B} = -NE(0) \qquad (6.32)$$

i.e., exactly equal to the energy lost by the particles.

Obviously, the results obtained [eqs.(6.31) and (6.32)] agree fully with the conclusions drawn previously [see especially eqs.(6.17) and (6.25)].

Footnote to p.15

Sazonov (Ref.14) calculated the tensor $R_{\alpha\beta\sigma\varepsilon}$ using a kinetic equation. This can also be done by the Epstein coefficient method [see, in part, Takanura (Ref.7)]. Moreover, it is important that the electron moving in the magnetic field with fixed E and $\theta$ emits, in a given direction and for a given frequency, a wave with completely defined (in the general case - elliptical) polarization. The given electron will absorb or intensify only that wave with a polarization it can emit (this conclusion is especially clear from the quantum theory since the probabilities of spontaneous emission, absorption, and induced radiation are proportional to the square of the module of one and the same parent element).

REFERENCES /54


1. Ginzburg, V.L. and Syrovatskiy, S.I.: The Origin of Cosmic Rays. Izv. Akad. Nauk SSSR, 1963. Supplementary English Translation, Pergamon Press, 1964.
2. Ginzburg, V.L. and Syrovatskiy, S.I.: Usp. Fiz. Nauk, Vol.87, p.65, 1965; Ann. Rev. Astron. Astrophys., Vol.3, p.297, 1965.
3. Vladimirskiy, V.V.: Zh. Eksperim. i Teor. Fiz., Vol.18, p.392, 1948.
4. Schwinger, J.: Phys. Rev., Vol.75, p.1912, 1949.
5. Westfold, K.C.: Astrophys. J., Vol.130, p.241, 1959.
6. Takanura, T.: Publ. Astron. Soc. Japan, Vol.12, p.352, 1960.
7. Kawabata, K.: Publ. Astron. Soc. Japan, Vol.16, p.30, 1964.
8. Ginzburg, V.L. and Ozernoy, L.M.: Radiofiz., Vol.9, p.221, 1966; Astrophys. J., Vol.144, p.599, 1966.
9. Zheleznyakov, V.V.: Zh. Eksperim. i Teor. Fiz., Vol.51, p.570, 1966; Astr. Zh., Vol.44, p.42, 1967.
10. McCray, R.: Science, Vol.154, p.1320, 1966.
11. Frank, I.M.: Usp. Fiz. Nauk, Vol.58, Part III, 1956.
12. Landau, L.D. and Lifshits, Ye.M.: Field Theory (Teoriya Polya). Izd. "Nauka", 1967.
13. Gradshteyn, I.S. and Ryzhyn, I.M.: Tables of Integrals. Fizmatgiz, 1962.
14. Sazonov, V.N. and Tsitovich, V.N.: Radiofiz., 1968; Sazonov, V.N.: Astron. Zh., 1968.
15. Wild, J.P., Smerd, S.F., and Weiss, A.A.: Ann. Rev. Astron. Astrophys., Vol.1, p.291, 1963; Usp. Fiz. Nauk, Vol.84, p.99, 1964.
16. Manley, O.: Interaction of Relativistic Particles with Magnetic Fields, Reprint, 1967; Abstract: Phys. Rev. Letters, Vol.18, 2A, No.13, 1967.
17. Ginzburg, V.L. and Syrovatskiy, S.I.: Proc. IAU Symposium, No.31, Holland, 1966. See also Usp. Fiz. Nauk, Vol.88, p.485, 1966.
18. Ginzburg, B.L.: The Propagation of Elementary Waves in Plasma. "Nauka", 1967.



Translated for the National Aeronautics and Space Administration by the O.W.Leibiger Research Laboratories, Inc.

/55

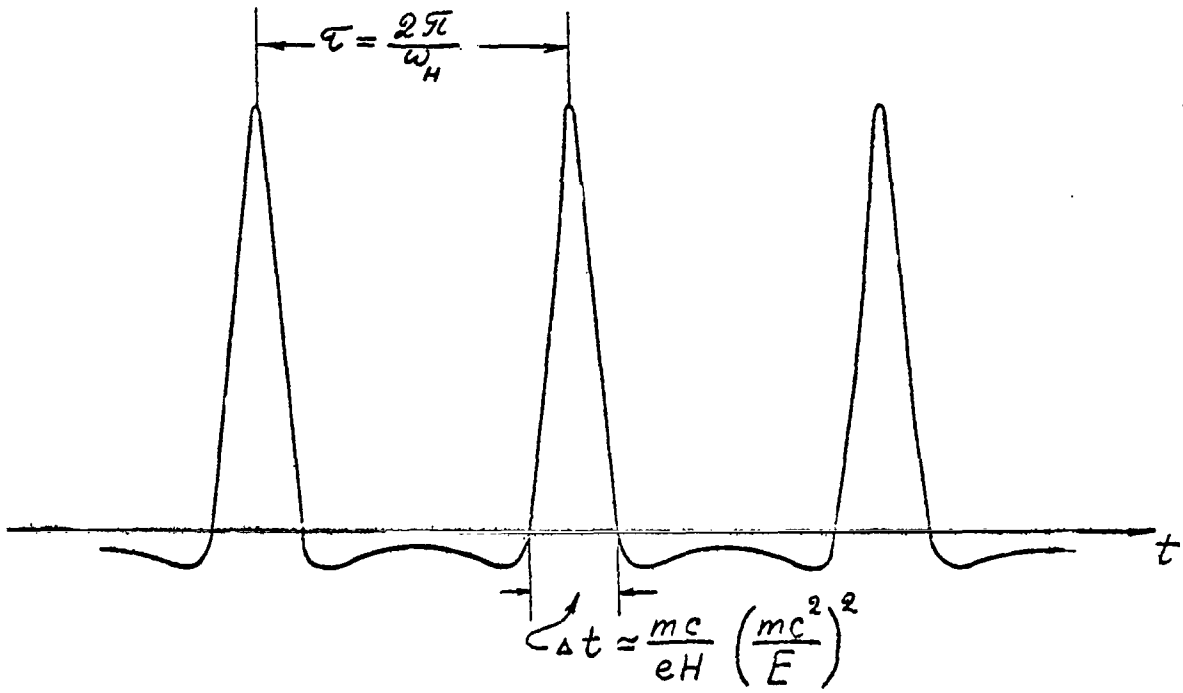


Fig.1

/56

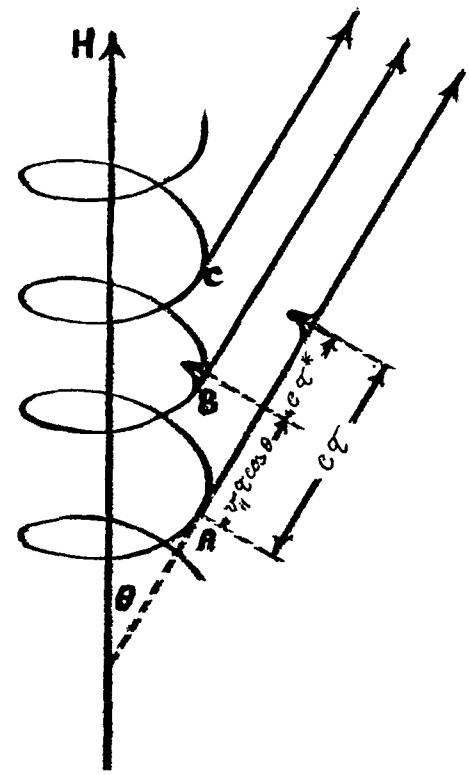


Fig.2

/57

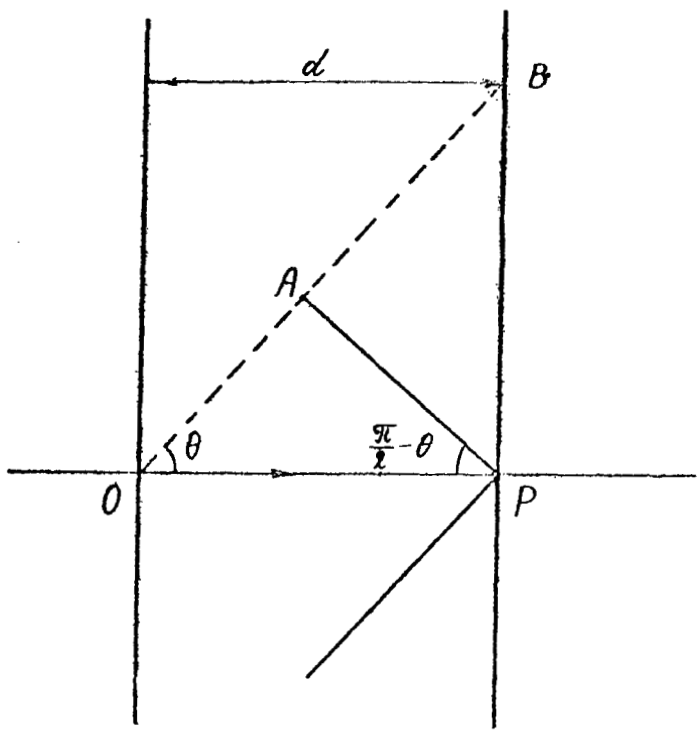


Fig.3

/58

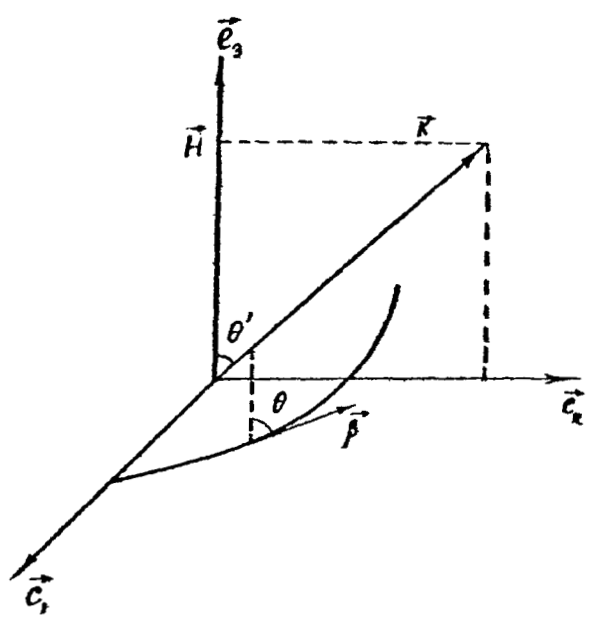


Fig.4

/59

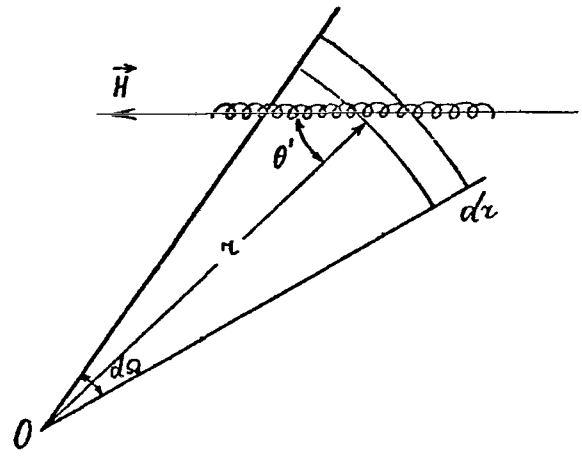


Fig.5

/60

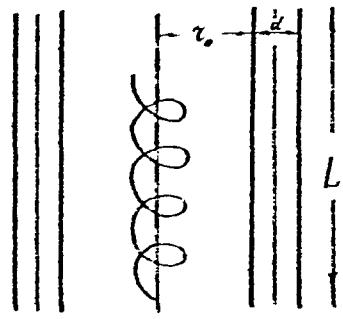


Fig.6

/61

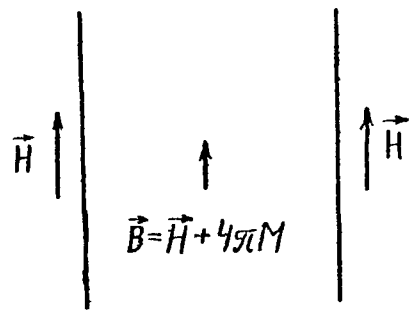


Fig.7

National Aeronautics and Space Administration
WASHINGTON, D. C.

OFFICIAL BUSINESS

FIRST CLASS MAIL

POSTAGE AND FEES PAID
NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION

040 001 50 51 3DS 00903
AIR FORCE WEAPONS LABORATORY/AFSC/
KIRTLAND AIR FORCE BASE, NEW MEXICO 87117

ATT MISS MADELINE F. CANOVA, CHIEF TECHNICA
LIBRARY /WLIL/

POSTMASTER: If Undeliverable (Section 158 Postal Manual) Do Not Return

*"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."*

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

# NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Notes, and Technology Surveys.

*Details on the availability of these publications may be obtained from:*

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546